Jan. 12-16, 2025

The 2nd Nuclear Physics Tortoise Lecture Series (NPTLS) 2026

Pairing in ultracold atoms and neutron star

Hiroyuki Tajima

The University of Tokyo, Japan

Self-introduction: **Hiroyuki Tajima**

<u>2012-2017</u> Keio Univ. (Ohashi group, cond-mat, Japan)

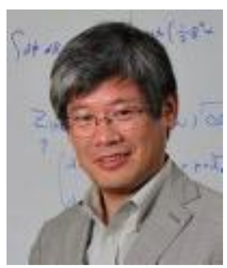
<u>2017-2019</u> RIKEN (Hatsuda group, nucl-th, Japan)

<u>2019-2021</u> Kochi Univ. (Iida group, nucl-th, Japan)

<u>2021-recent</u> Univ. Tokyo (Liang group, nucl-th, Japan)

Prof. Y. Ohashi

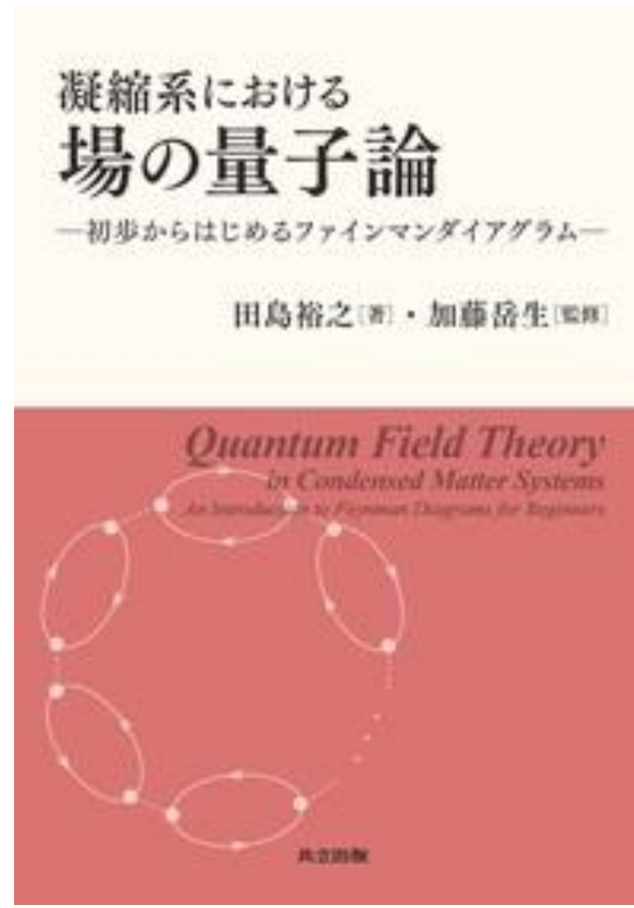
Prof. T. Hatsuda



Prof. K. Iida

Prof. H. Liang

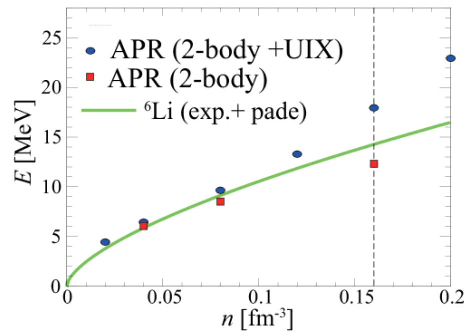
Textbook in Japanese (Aug. 2025)



https://www.kyoritsu-pub.co.jp/book/b10136511.html

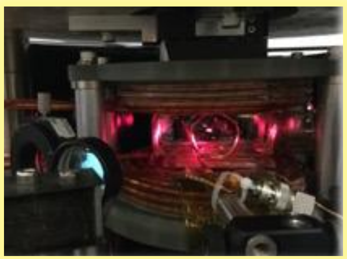
Self-introduction: **Hiroyuki Tajima**
Ultracold atoms to neutron star matter

Unitary Fermi gas/ BEC-BCS crossover



PRX, **7**, 041004 (2017).

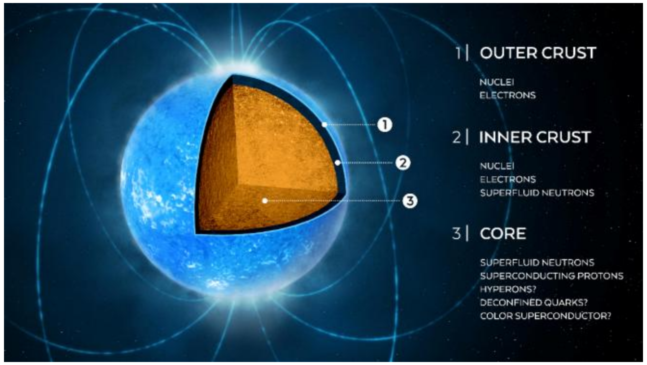
Ultracold atoms



- Tunable interaction
- Boson/fermion
- Gas/lattice
- (pseudo) spin and color

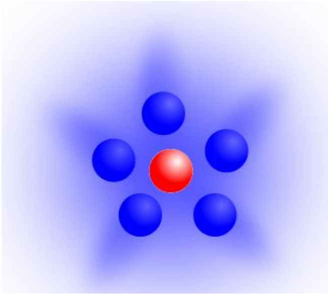
Photo: http://www.sci.osaka-cu.ac.jp/phys/laser/research_Li.html

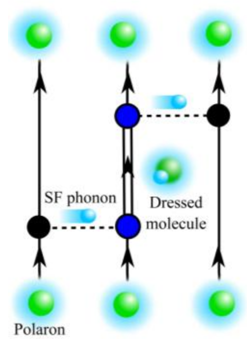
Neutron star matter



A. L. Watts, *et al.*, RMP **88**, 021001 (2016).

Polarons





PLB **851**, 138567 (2024).
arXiv:2505.19117

More (but not too) "macroscopic" physics between ultracold atoms and neutron stars

arXiv:2007.00926

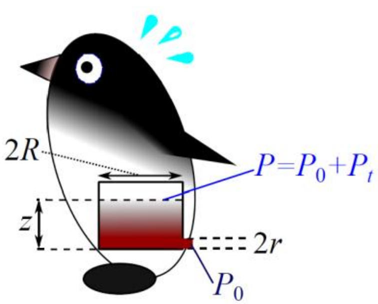
(Published in Bulletin of Kochi Univ.)

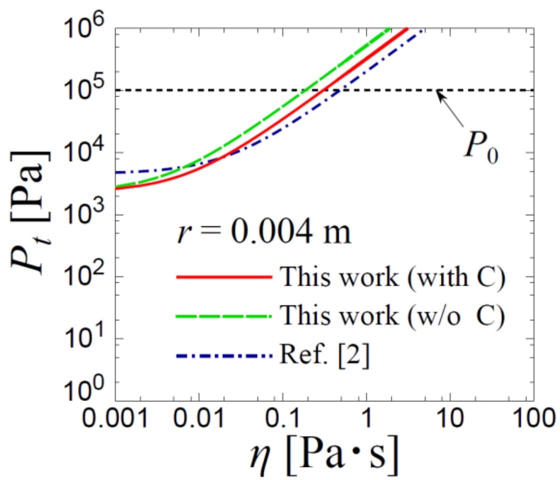
Projectile Trajectory of Penguin's Faeces and Rectal Pressure Revisited

Hiroyuki Tajima[1] and Fumiya Fujisawa[2]

[1]*Department of Mathematics and Physics, Kochi University, Kochi 780-8520, Japan*
[2]*Katsurahama Aquarium, Kochi 781-0262, Japan*

(Dated: July 1, 2020)



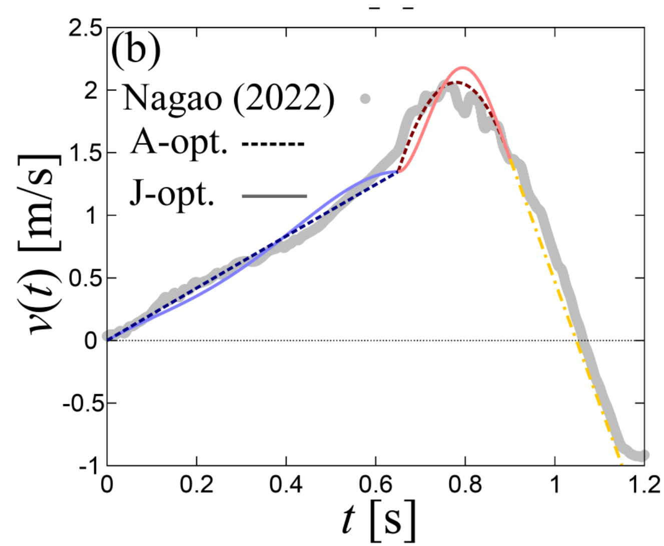


arXiv:2510.19841

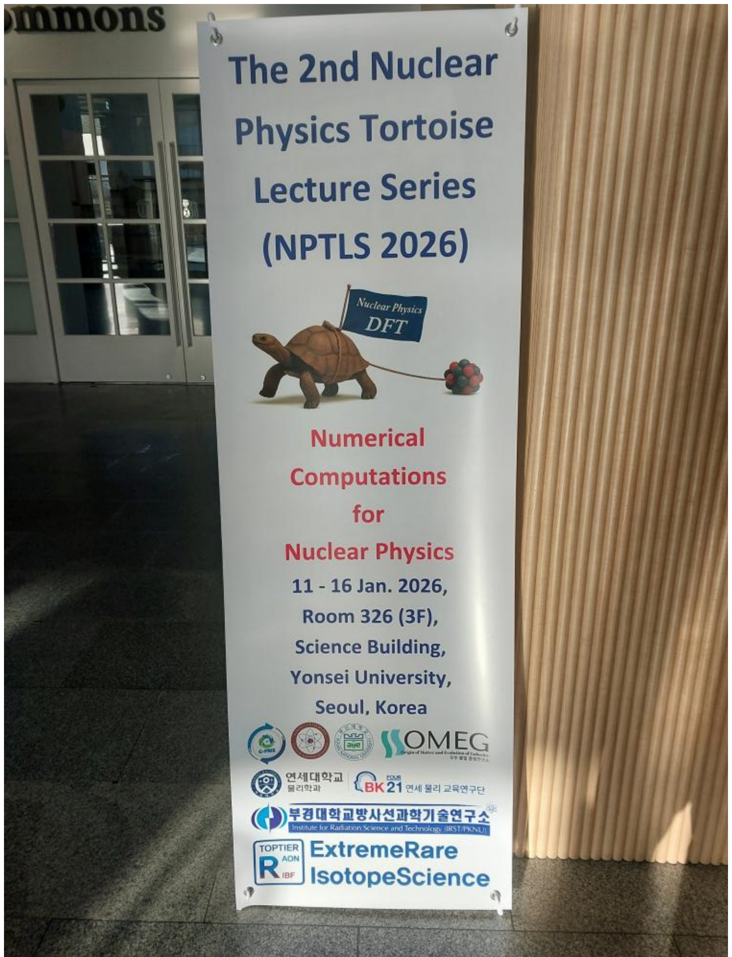
(submitted to Phys. Rev. E)

Optimal control approach to Olympic weightlifting exercise: Minimal model of the snatch pull

Hiroyuki Tajima,[1,2,3] Hideyuki Nagao,[4] Kenya Tanaka,[5] Hideaki Nishikawa,[6,7] Eishiro Murakami,[8] Akito Ida,[9] and Masataka Watanabe[1]



I am sorry that the latter half is a little bit different topic from DFT



Prof. H. Liang

Could you give a lecture instead of me? (On **25th Dec. 2025**)

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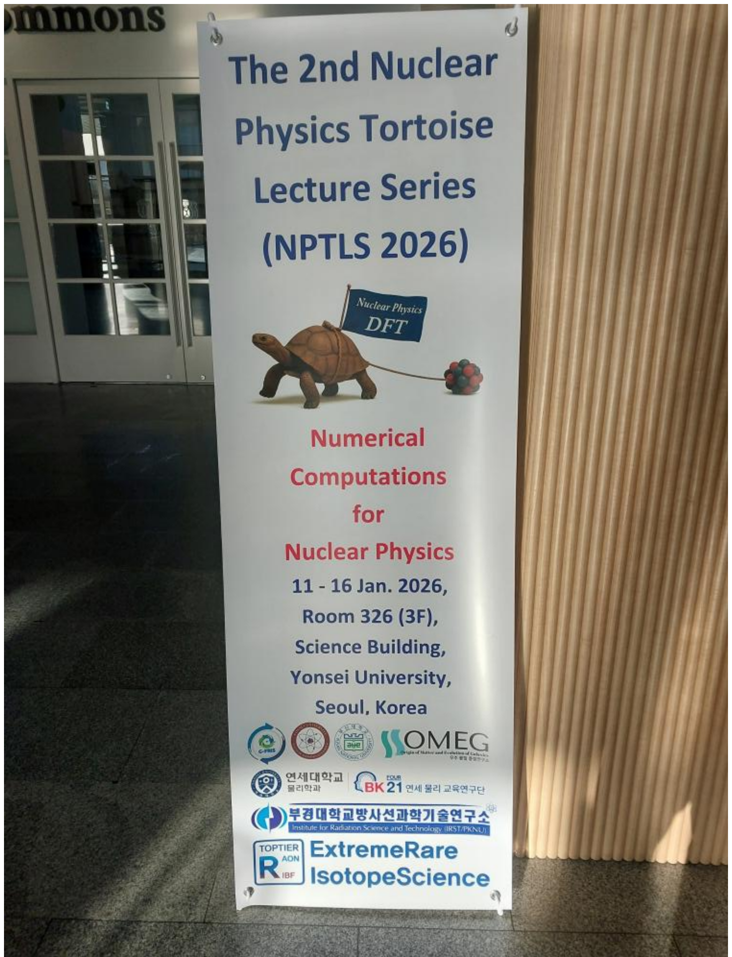


Prof. H. Liang

Xmas gift!!!

Could you give a lecture instead of me? (On **25th Dec. 2025**)

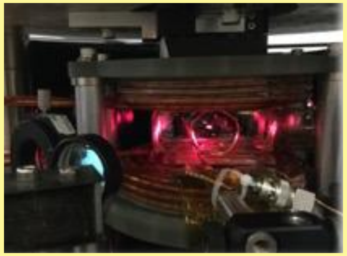
Plan of the lecture

- Bulk nuclear matter from a perspective of ultracold atomic physics
- What is an ultracold atom? Why is it useful?
- What is an outcome to nuclear physics from ultracold atom physics?

1. Introduction to ultracold atom as a quantum simulator of nuclear systems
2. Ultracold Fermi gas and renormalization
3. Exercise 1: BCS-Eagles-Leggett theory for the BCS-BEC crossover
4. Hands-on exercise of mean-field description
5. Pairing fluctuations beyond mean-field theory
6. Exotic superfluid and nucleon superfluid
7. Exercise 2: Minimal framework for critical-temperature curve
8. Hadron-quark crossover from an ultracold-atom perspective

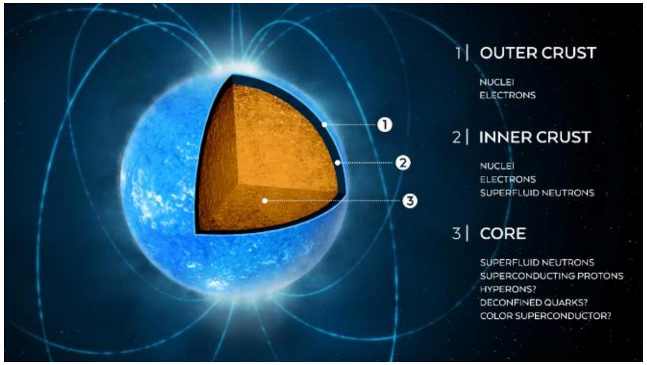
Part 1
-Introduction to ultracold atoms as a quantum simulator of nuclear systems-

Ultracold atoms



?

Neutron star matter



A. L. Watts, *et al*., RMP **88**, 021001 (2016).

➡ What is an ultracold atoms? Useful for nuclear physics?

Outline

- Nuclear matter and neutron star
- Ultracold atom as a “Toy model” for nuclear system
- Ultracold Fermi gas and pure neutron matter
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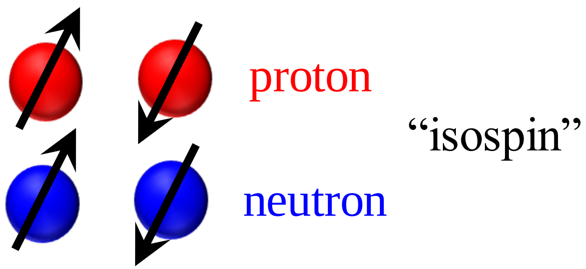
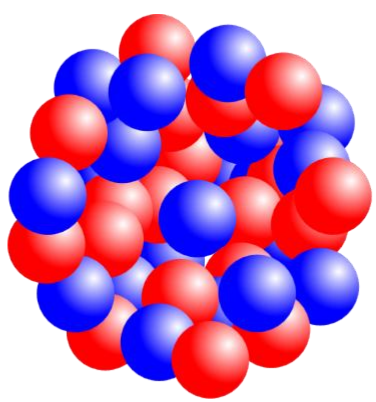
Nuclear many-body problem

- Many-body system consisting of neutrons and protons

Nucleon (spin-1/2 fermion)



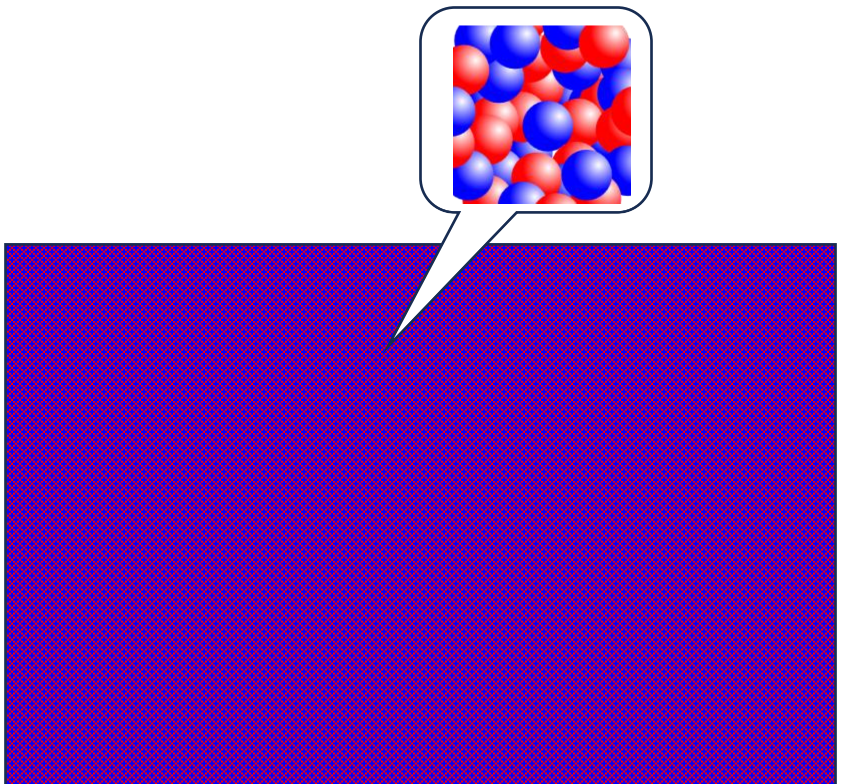
Nuclei (finite-size system)



$\rho_0 = 0.16\ \mathrm{fm}^{-3}$: nuclear saturation density

Nuclear matter

usually infinite size, homogeneous



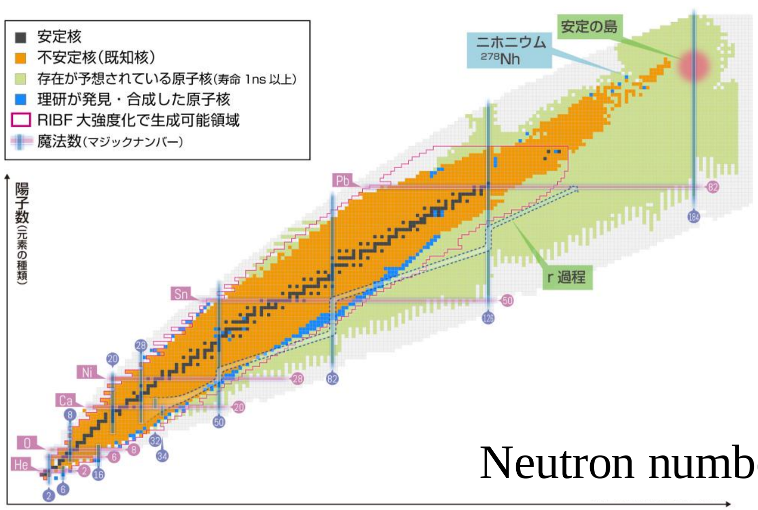
Neutron star

- “Super-gigantic nuclei” with radius 10km and 2 solar mass

Nuclear matter is directly relevant to the internal structure of neutron stars

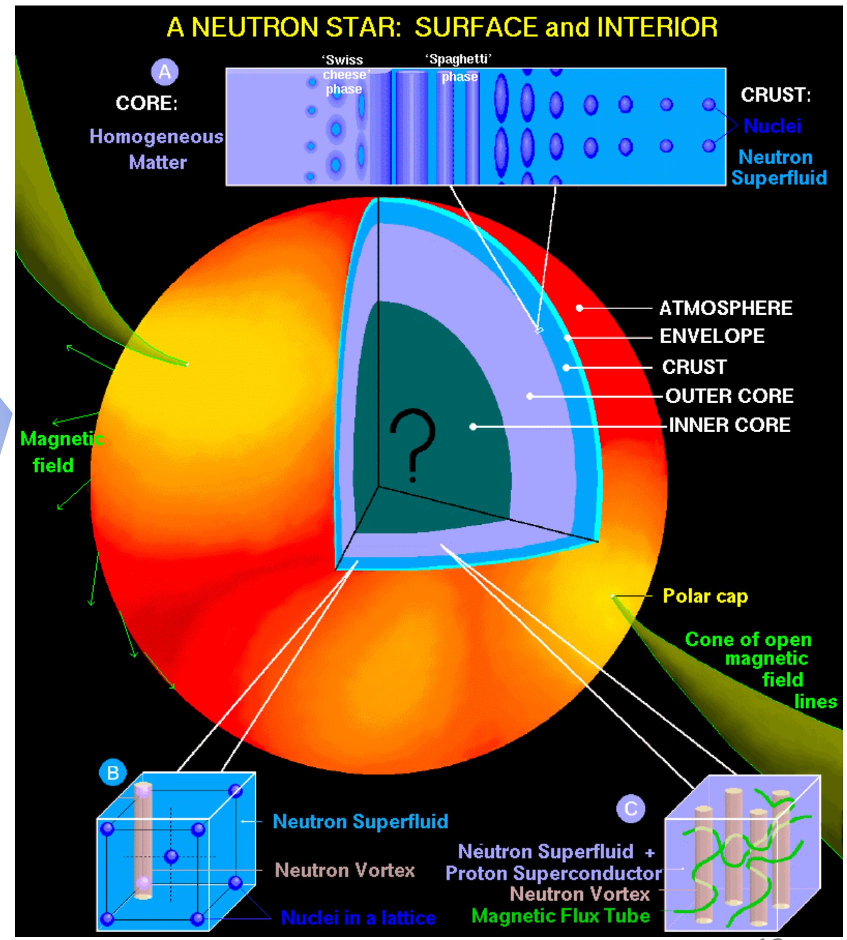
Nuclear chart

Proton number



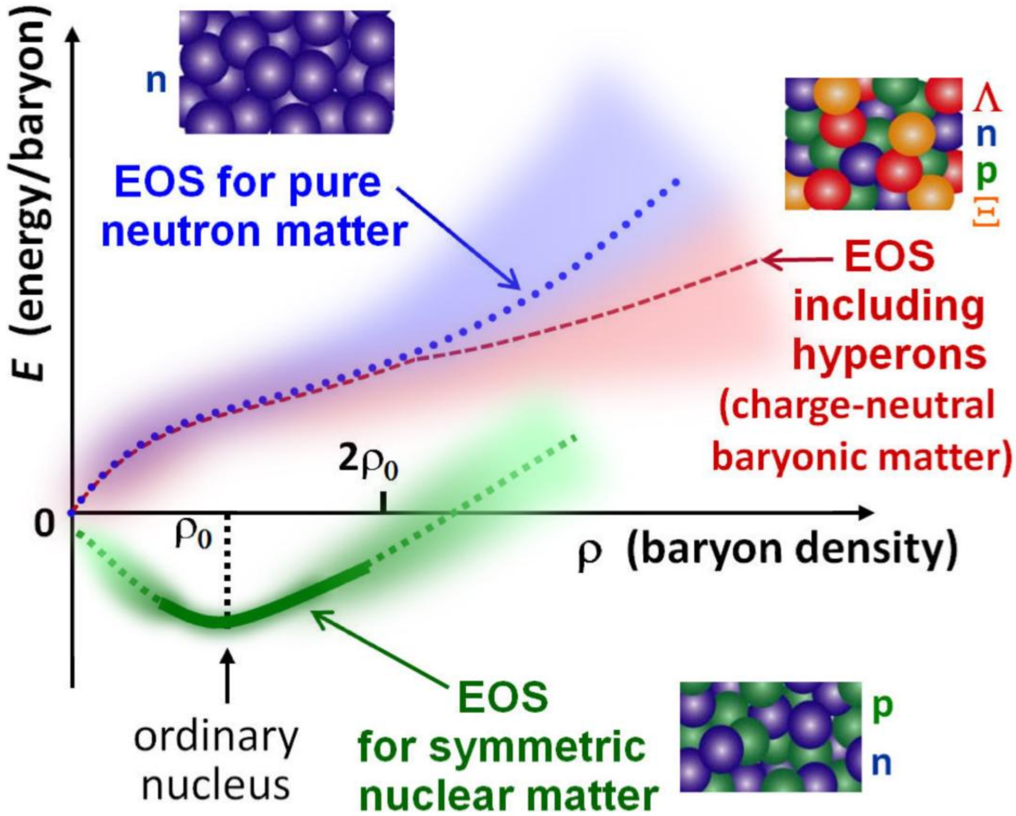
Neutron number

https://www.nishina.riken.jp/researcher/archive/illust.html



Created by Dany Page: https://www.astroscu.unam.mx/neutrones/NS-Picture/NStar/NStar.html

Nuclear matter equation of state



H. Tamura, JPS Conf. Proc. **1**, 011003 (2014).

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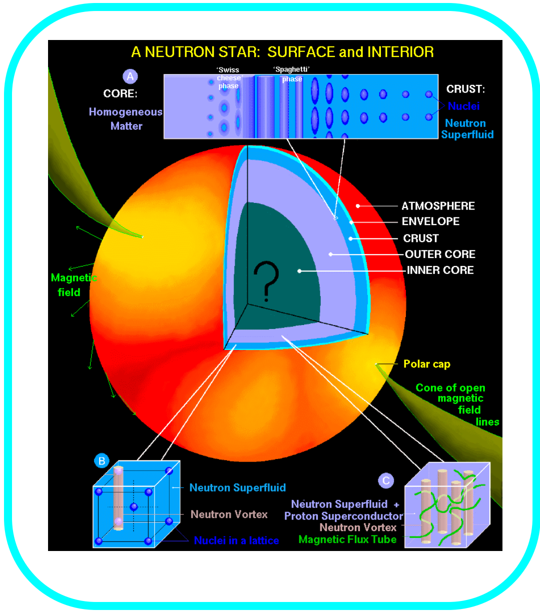
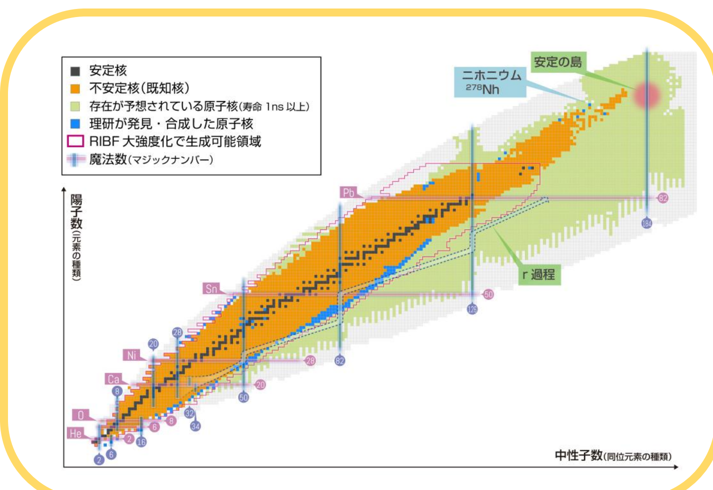
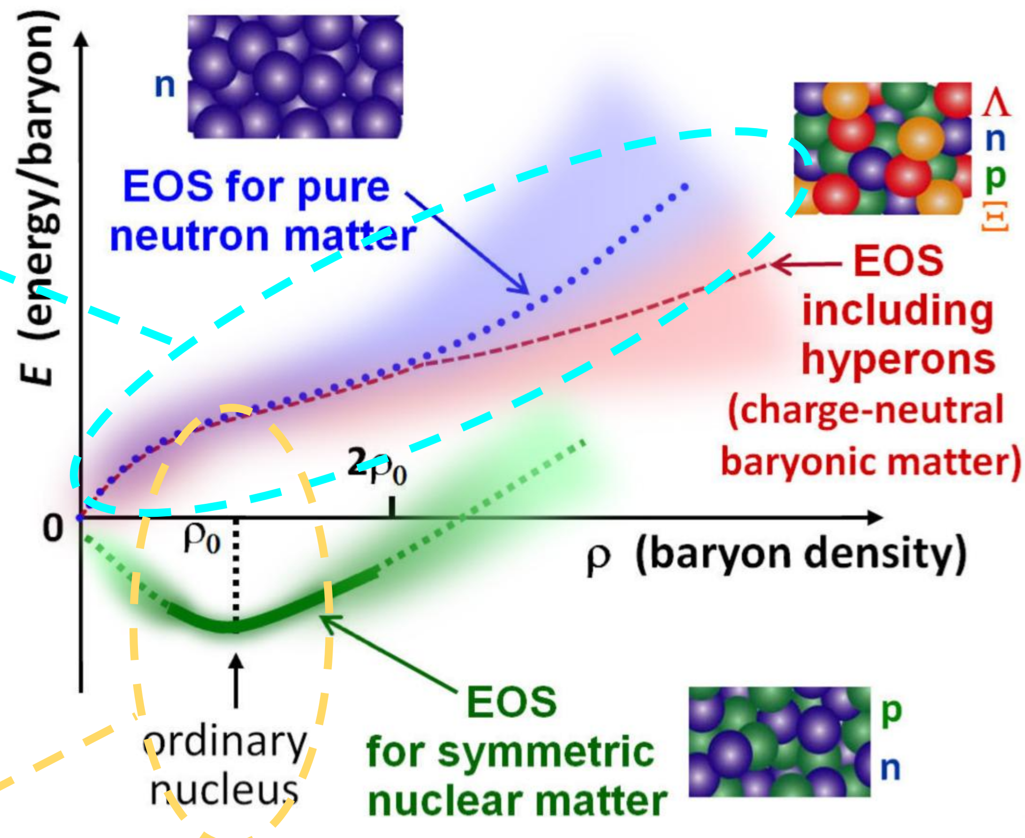
Nuclear matter equation of state







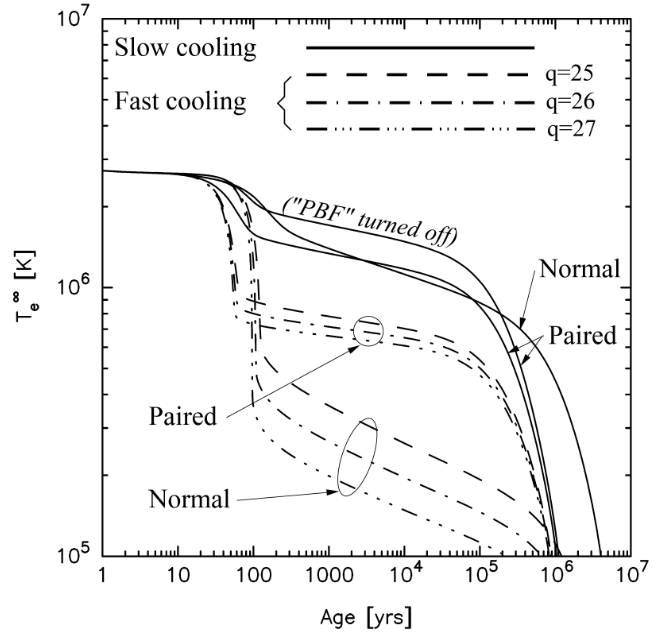
H. Tamura, JPS Conf. Proc. **1**, 011003 (2014).

$\rho_0 = 0.16\ \mathrm{fm}^{-3}$: nuclear saturation density

Superfluid in a neutron star

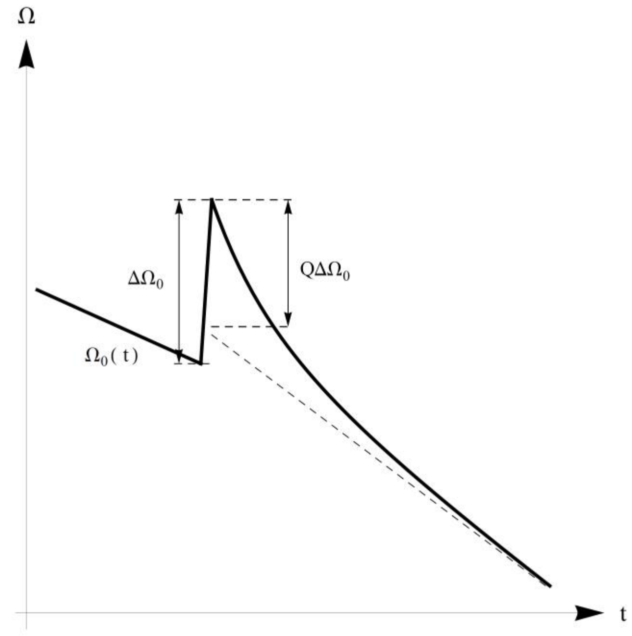
Nucleon superfluids affect the observed phenomena

Cooling Curve



D. Page, Fifty Years of Nuclear BCS 324-447 (2013).

Pulsar Glitch



van Eysden, C. A. (2011), PhD thesis, The University of Melbourne

Superfluid in a neutron star

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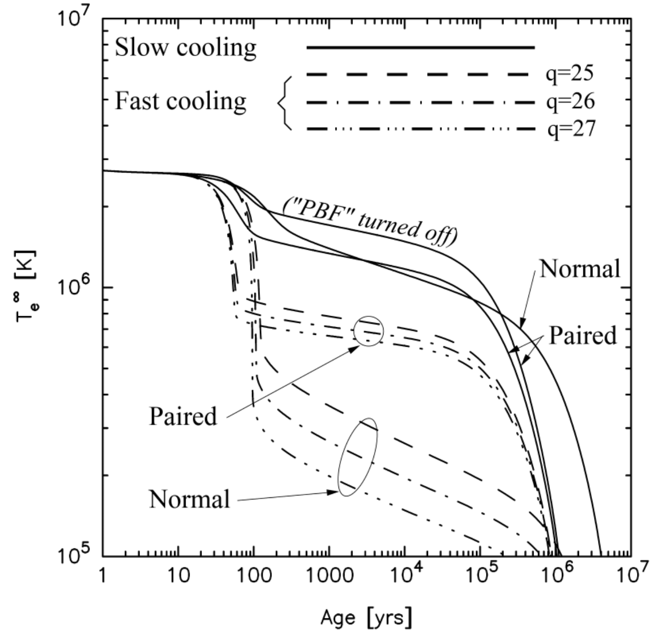
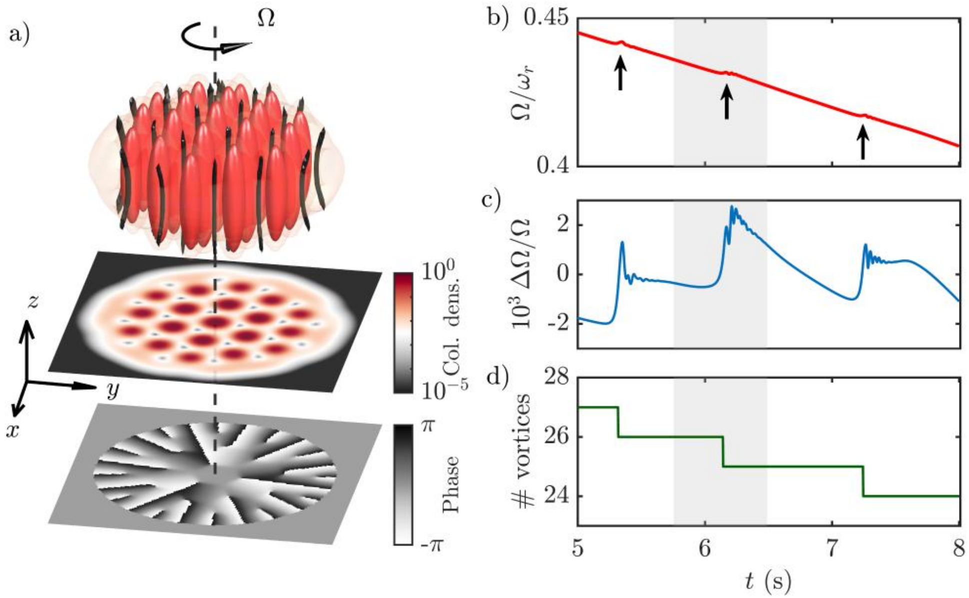


D. Page, Fifty Years of Nuclear BCS 324-447 (2013).

Glitch in "ultracold atoms"

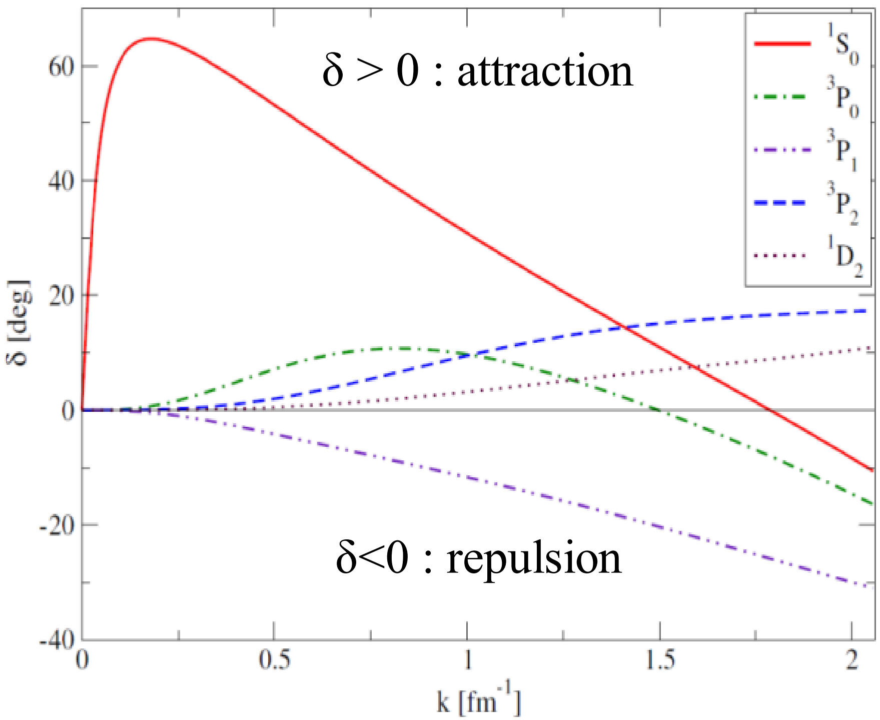


E. Poli, et al, Phys. Rev. Lett. **131**, 223401 (2023)

Nucleon superfluidity

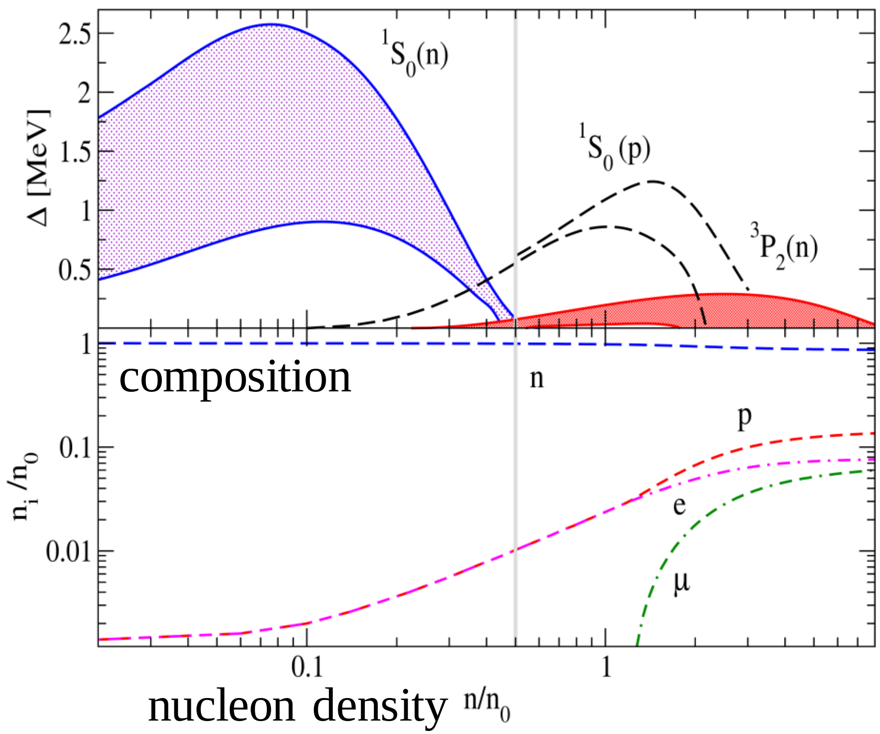
Nucleons (neutrons and protons) may exhibit the BCS pairing due to the attractive nuclear force

Phase shift of *NN* scattering



A. Gezerlis, *et al*, arXiv:1406.6109v2

Nucleon pairing gaps in unpolarized matter



Eur. Phys. J. A (2019) **55**: 167

Nucleon superfluidity

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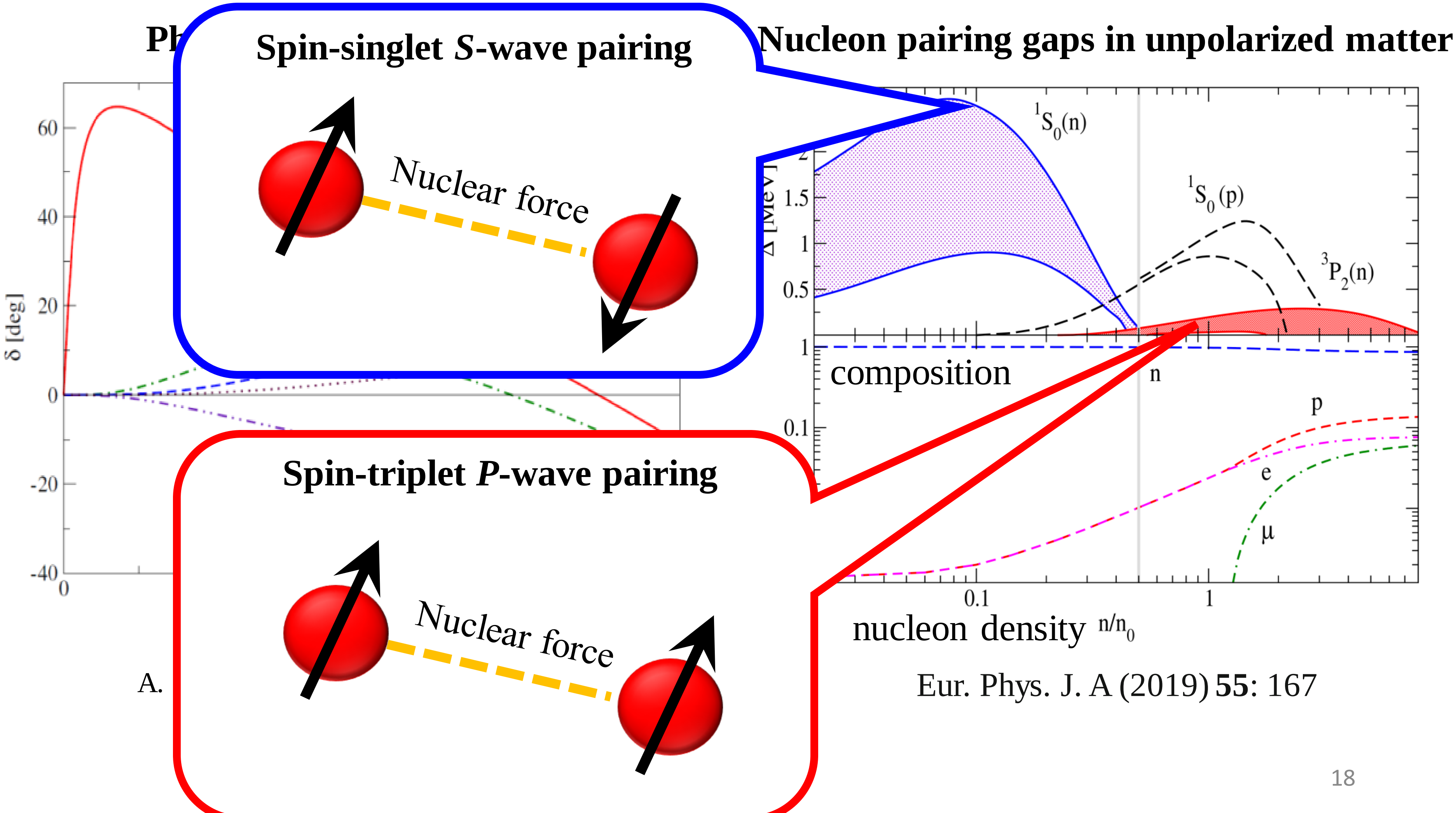
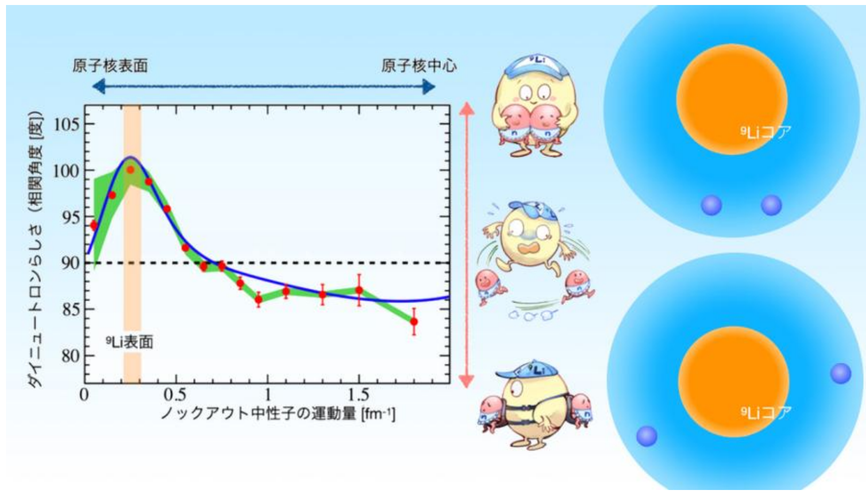


Eur. Phys. J. A (2019) **55**: 167

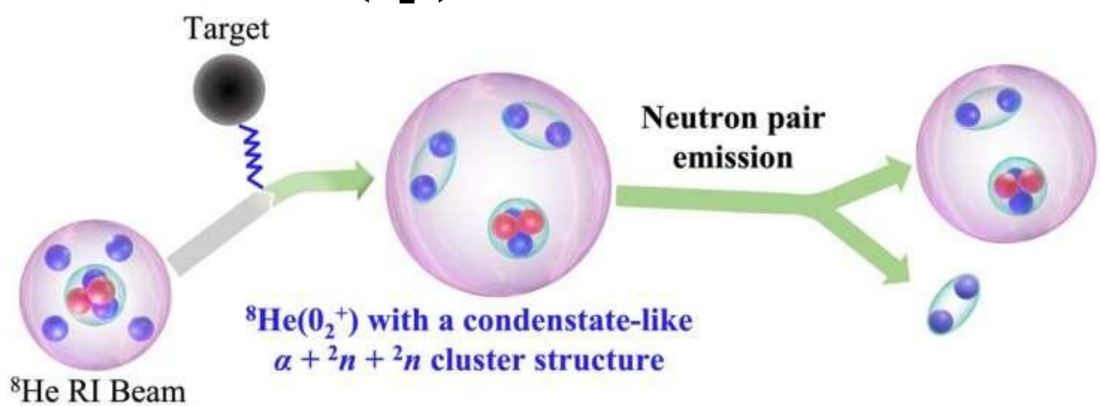
Strong dineutron correlations in neutron-rich nuclei

Surface Localization of the Dineutron in ^{11}Li



Y. Kubota et al., Phys. Rev. Lett. **125**, 252501 (2020)

Dineutron cluster in ^{8}He (0_2^+)



Z. H. Yang, et al., Phys. Rev. Lett. **131**, 242501 (2023)

Outline

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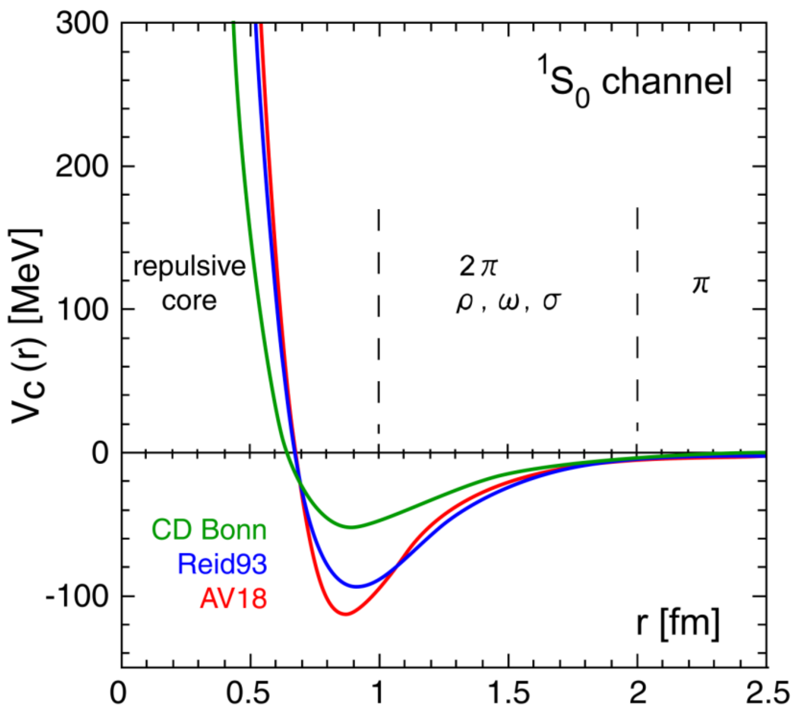
Infinite nuclear matter as a toy model for nuclei

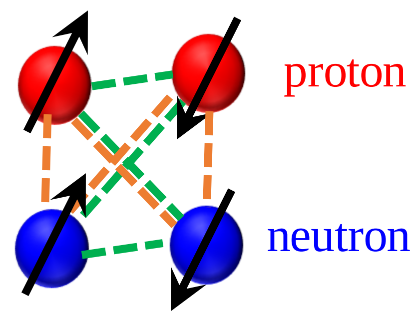
- "Toy model", but not so easy…

Nuclear force

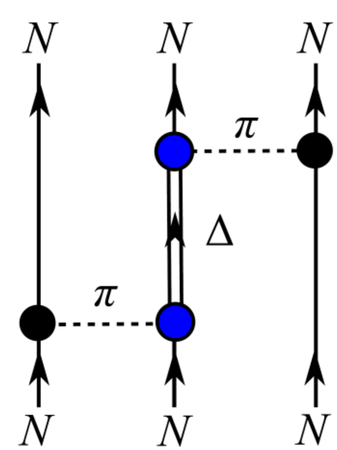


PRL **99,** 022001 (2007).

Isovector and isoscalar interactions



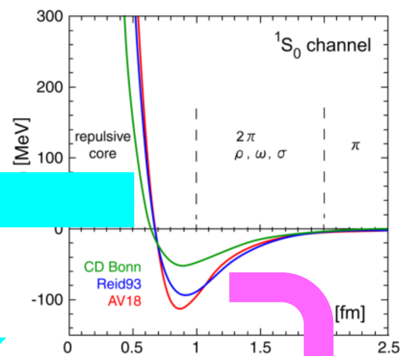
Three-body force



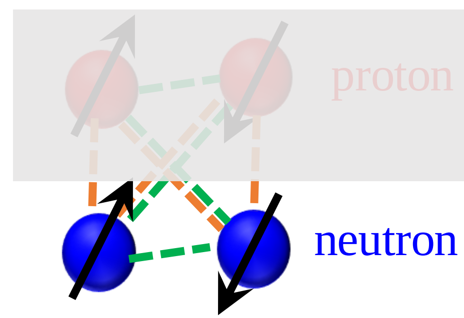
Is it similar to fermions with contact interaction?

- “Toy model”, but not so easy…

Nuclear force

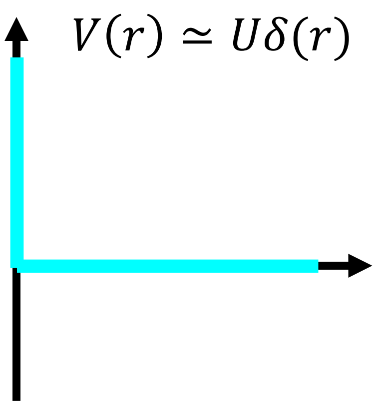


Isovector and isoscalar interactions



Short-range repulsion

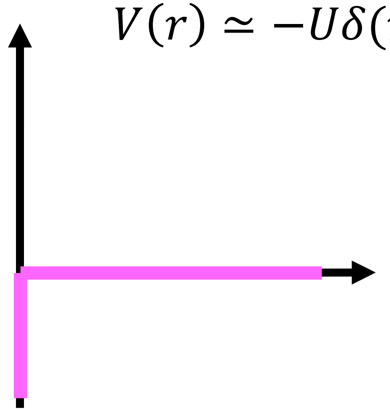
$V(r) \simeq U\delta(r)$



Dense case

Short-range attraction

$V(r) \simeq -U\delta(r)$

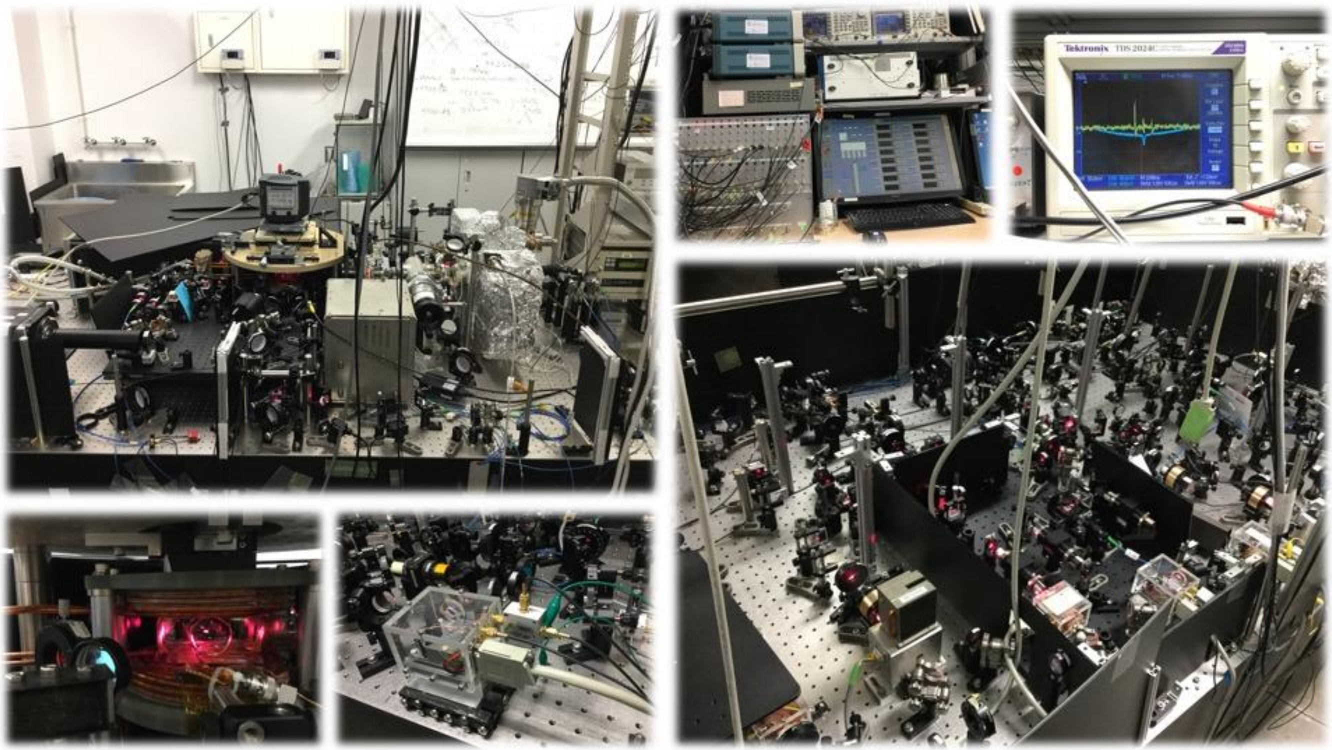


Dilute case

Ultracold Fermi gases
-Toy model for toy model-

…But various tunable parameters and many observables in experiments

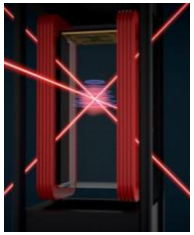
From Ultracold Quantum Gas Lab in Osaka Metro. Univ.



https://www.omu.ac.jp/sci/laserQ/resli/index.html

Solid Lithium

Oven
$T \gtrsim 600\mathrm{K}$



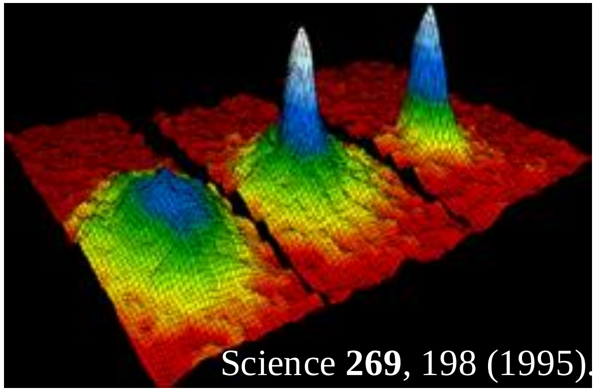
Laser Cooling

Credits: NASA/iGoal Animation

$T < 100\mathrm{nK}$

Bose and Fermi atomic gases

Velocity distribution in Bose-Einstein condensate



Science **269**, 198 (1995).

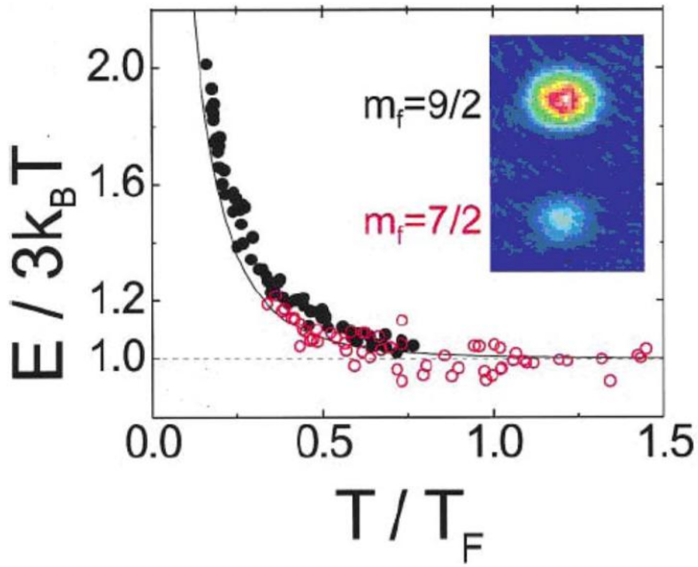
Nobel Prize in Physics 2001

E. A. Cornell

C. E. Wieman

W. Ketterle

Fermi pressure in quantum degenerate gases



Phys. Rev. Lett. **86**, 5409 (2001).

Bose atoms (^{7}Li, ^{87}Rb, ^{174}Yb,…)

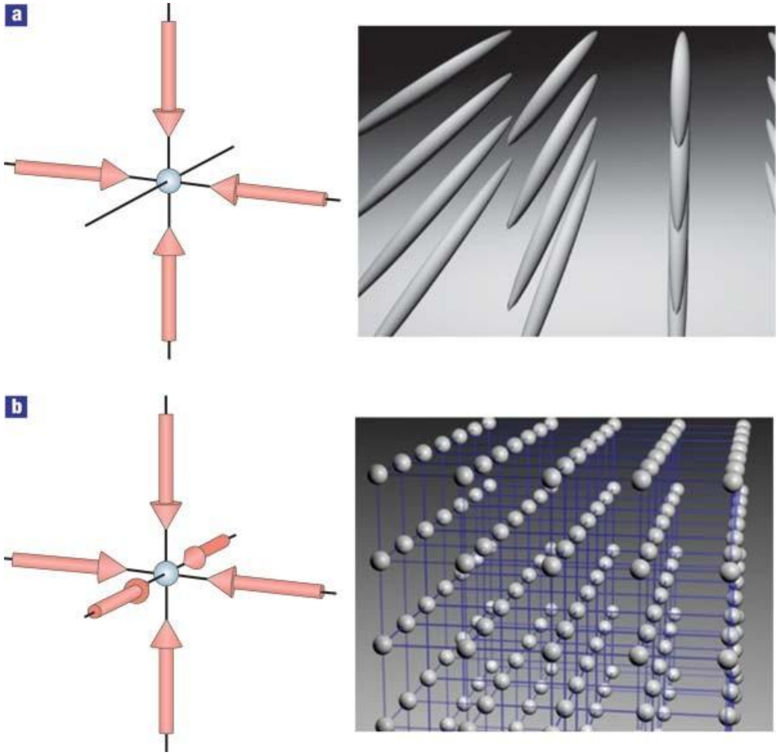
^{7}Li = 3 protons + 4 neutrons + 3 electrons
= (10 fermions)

Fermi atoms (^{6}Li, ^{40}K, ^{173}Yb,…)

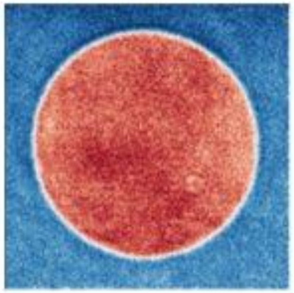
^{6}Li = 3 protons + 3 neutrons + 3 electrons
= (9 fermions)

Low dimension and lattice geometry

Optical lattice and 1D tube



I. Bloch, Nat. Phys. **1**, 23 (2005).



EDITORS' SUGGESTION

Two-Dimensional Homogeneous Fermi Gases

A homogeneous ultracold 2D Fermi gas is trapped in a box potential, which has advantages over harmonically trapped ones for probing strongly interacting systems.

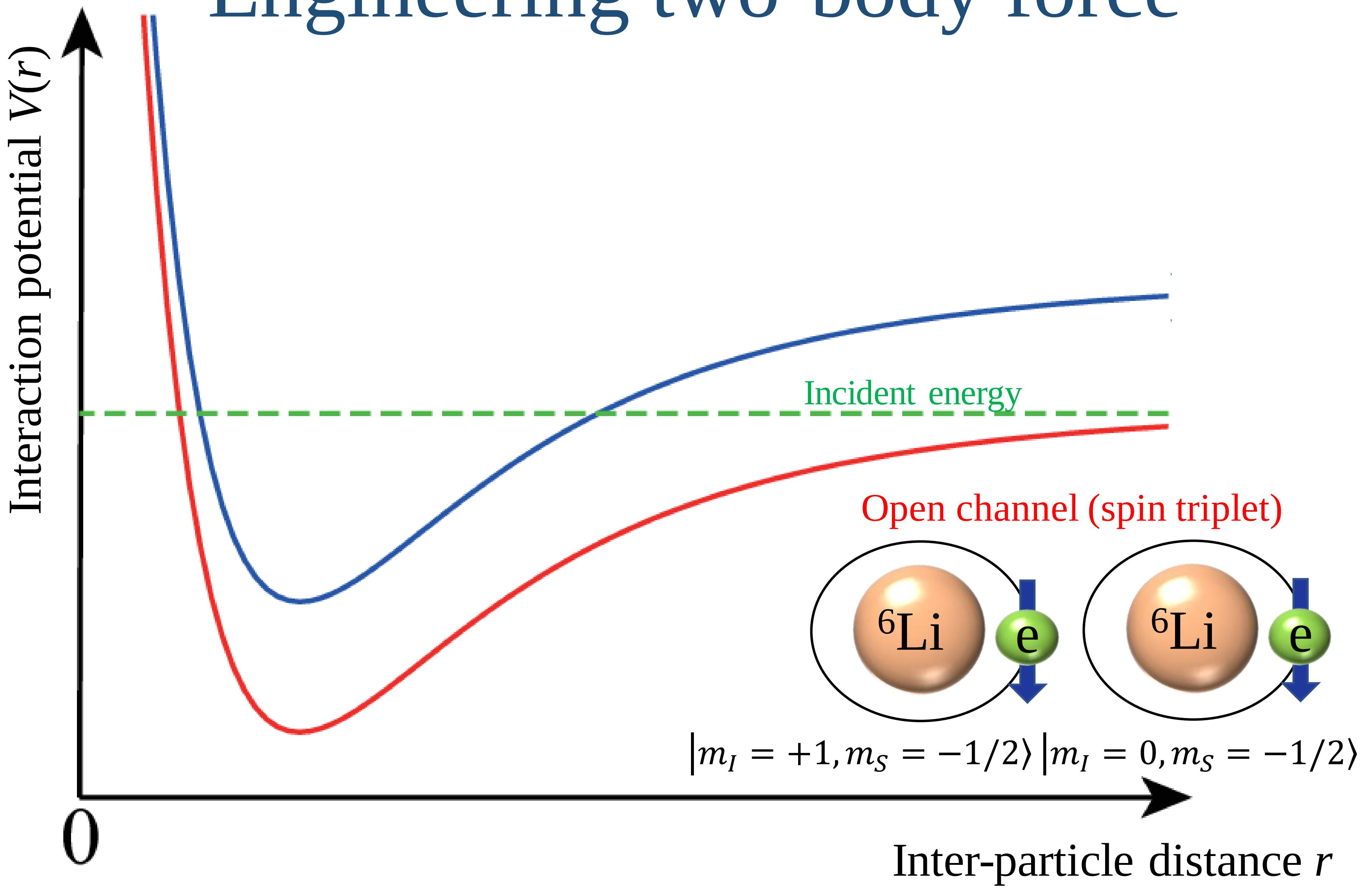
Klaus Hueck *et al.*

Phys. Rev. Lett. **120**, 060402 (2018)

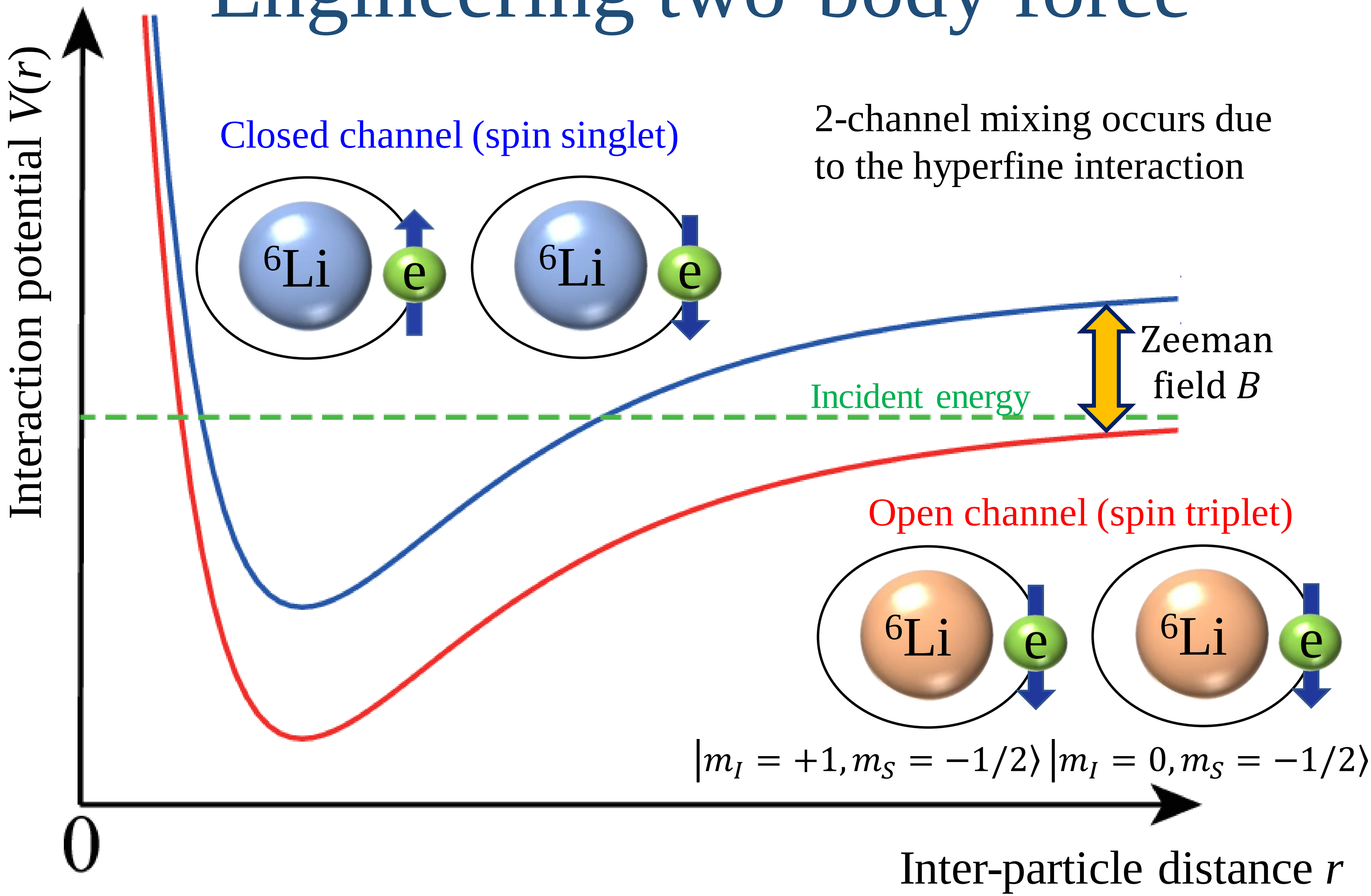
High-resolution trap potential

http://quantumgases.lens.unifi.it/exp/li

Fano-Feshbach resonance
Engineering two-body force

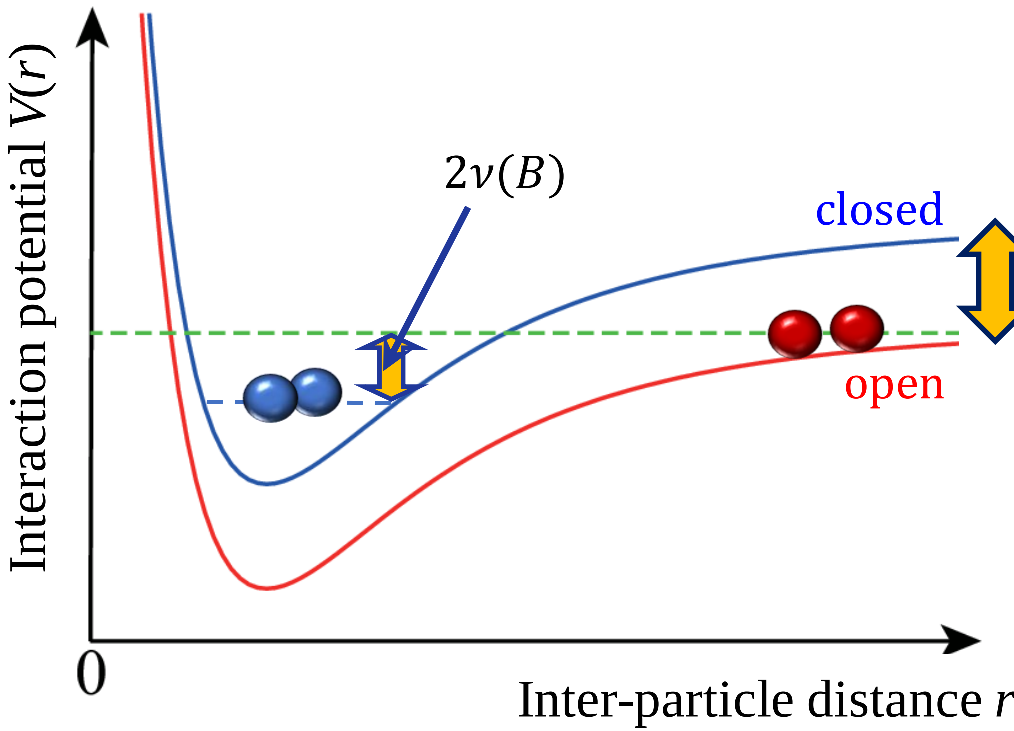


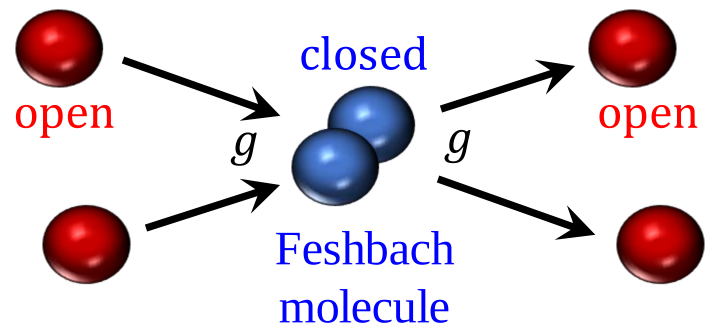
Fano-Feshbach resonance
Engineering two-body force



Fano-Feshbach resonance
Engineering two-body force

Atoms interacts with each other via an intermediate state
⇒attraction can be tuned by an external magnetic field



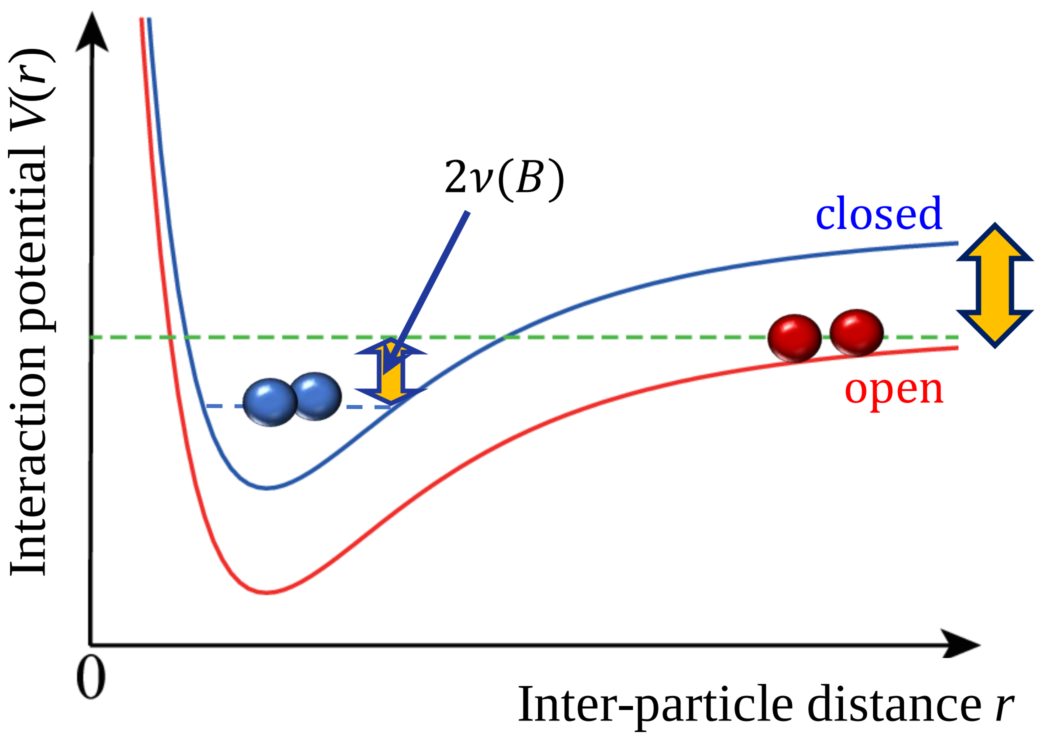


Fano-Feshbach resonance
Engineering two-body force

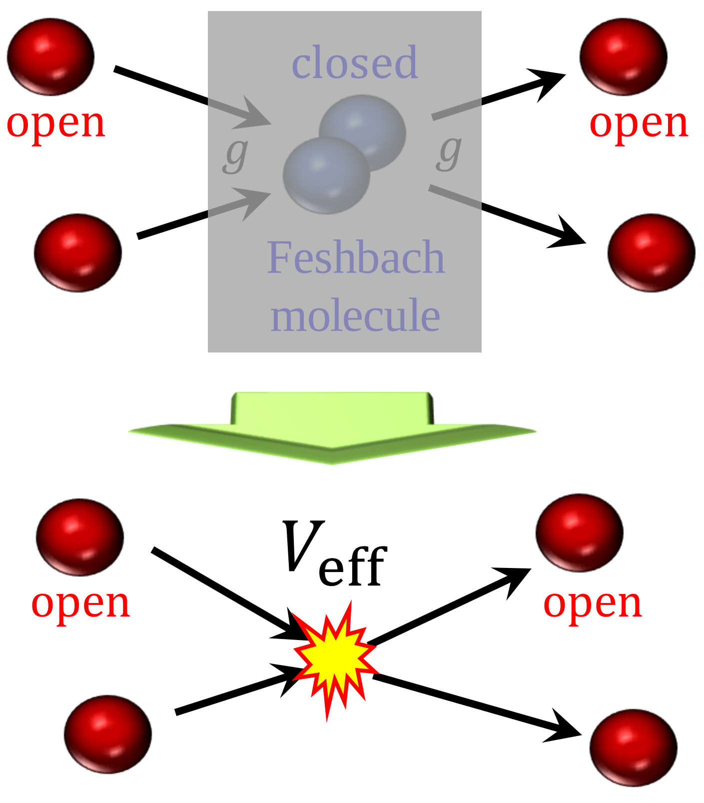
Atoms interacts with each other via an intermediate state
⇒attraction can be tuned by an external magnetic field

Effective attraction

$$V_{\mathrm{eff}} \simeq -\frac{g^2}{2\nu(B)}$$

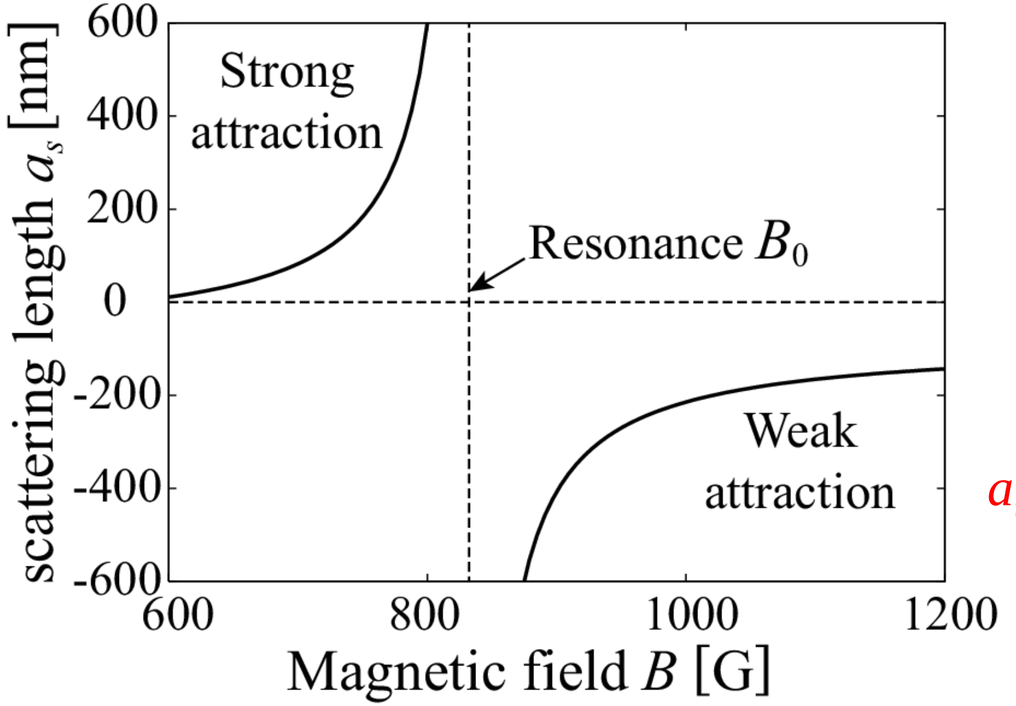


"Mediator" of effective interaction



Fano-Feshbach resonance
Engineering two-body force

Feshbach resonance in a ^{6}Li Fermi gas



***B*-dependence near Feshbach resonance**

$$a_s(B) = a_{\mathrm{bg}}\left(1 + \frac{W_{\mathrm{res}}}{B - B_0}\right)$$

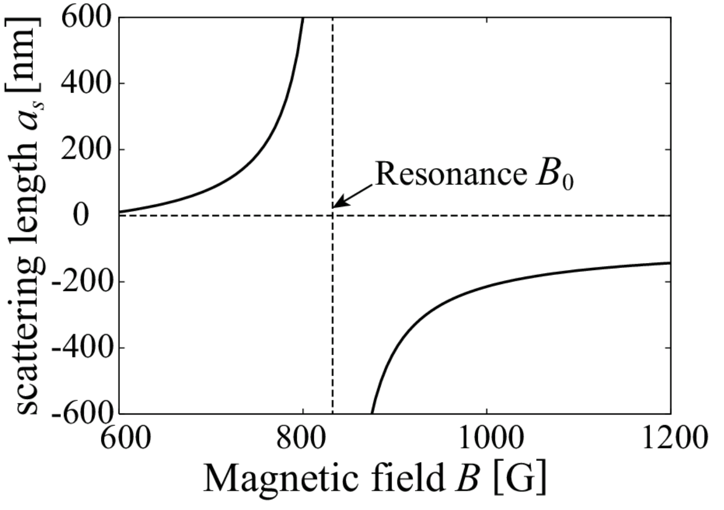
Resonance width $W_{\mathrm{res}} = 262.3\mathrm{G}$

Background scattering length $a_{\mathrm{bg}} = -1582a_0$

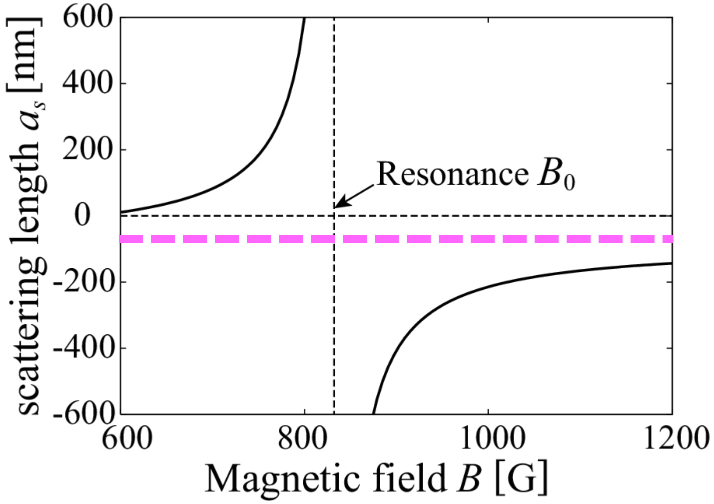
a_s can be controlled precisely by tuning B

G. Zürn, *et al*., PRL **110**, 135301 (2013).

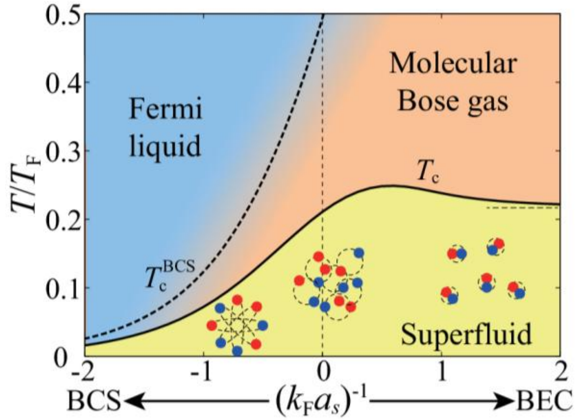
Tunable interaction near the Fano-Feshbach resonance



Tunable interaction near the Fano-Feshbach resonance



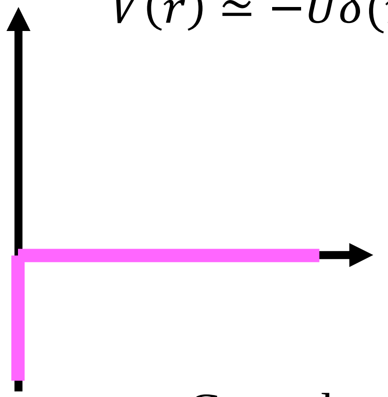
Pairing **due to attraction**



Y. Ohashi, HT, and P. van Wyk, Prog. Part. Nucl. Phys. **111**, 103739 (2020).

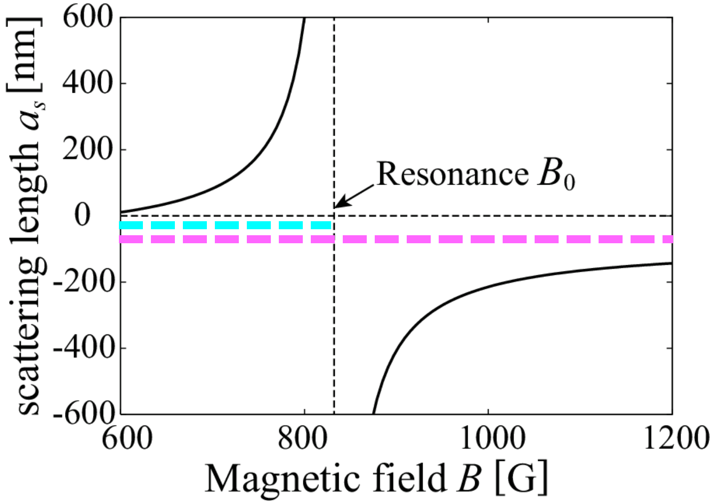
Short-range attraction

$$V(r) \simeq -U\delta(r)$$

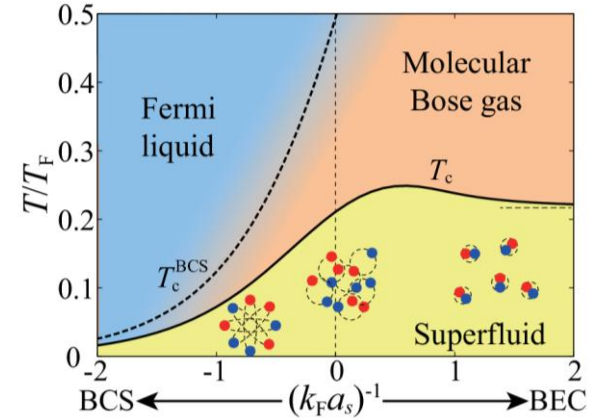


Ground state

Tunable interaction near the Fano-Feshbach resonance



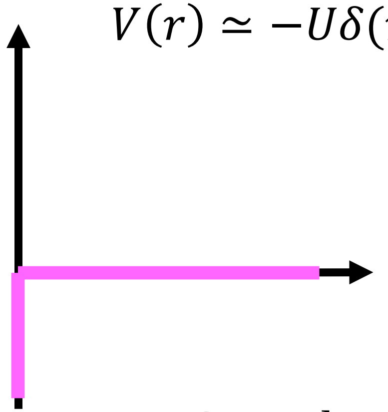
Pairing due to attraction



Y. Ohashi, HT, and P. van Wyk, Prog. Part. Nucl. Phys. **111**, 103739 (2020).

Short-range attraction

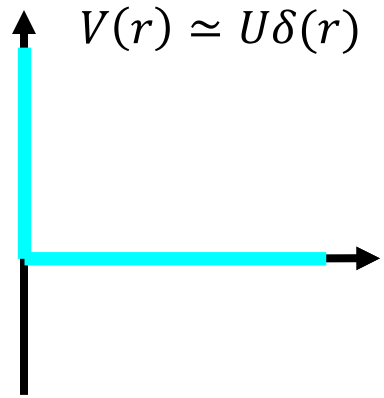
$V(r) \simeq -U\delta(r)$



Ground state

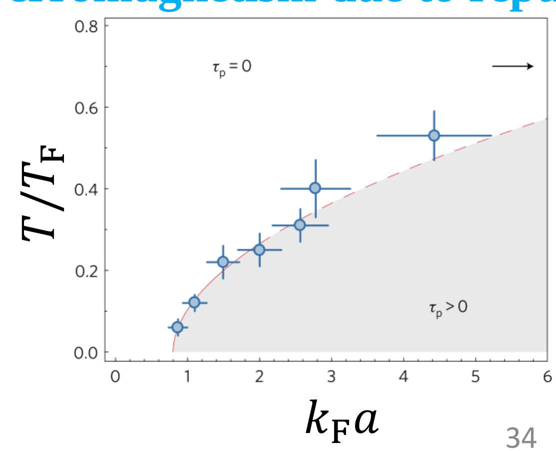
Short-range repulsion

$V(r) \simeq U\delta(r)$



Excited state (a>0)

Ferromagnetism due to repulsion

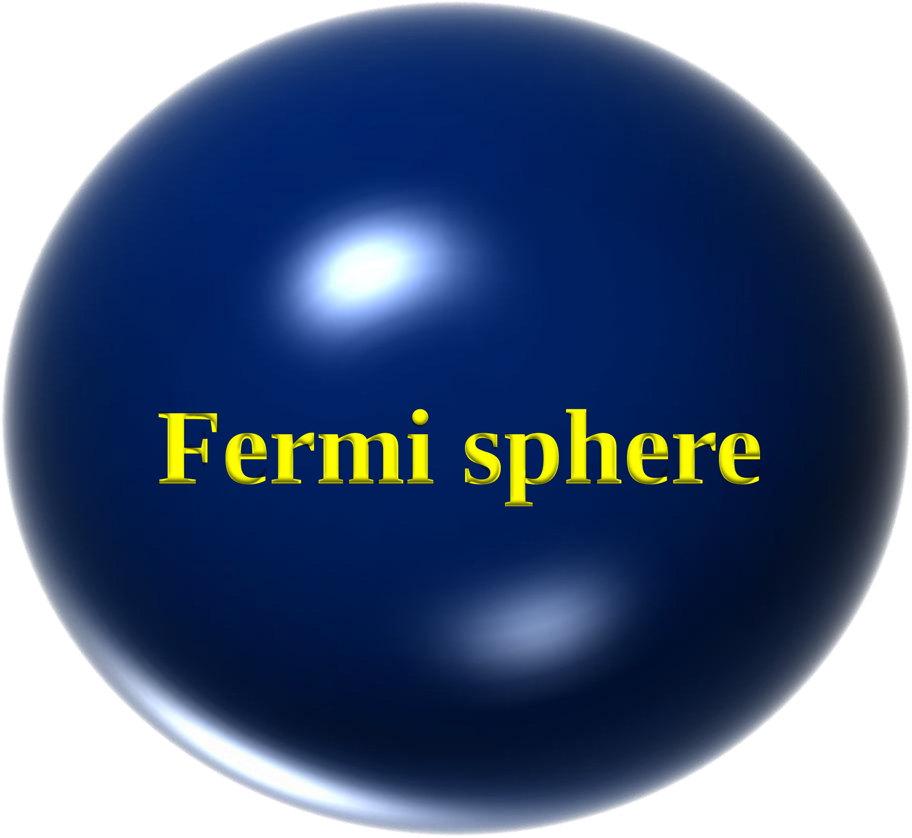


G. Valtolina, et al., Nat. Phys. **13**, 704 (2017).

Ground-state of Fermi systems

Electrons, neutron, proton, quark, etc…

Ideal Fermi gases



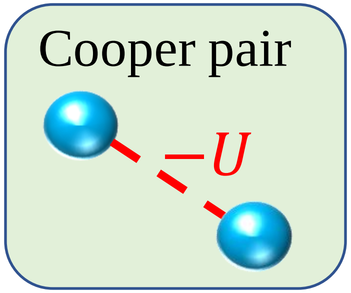
Momentum space

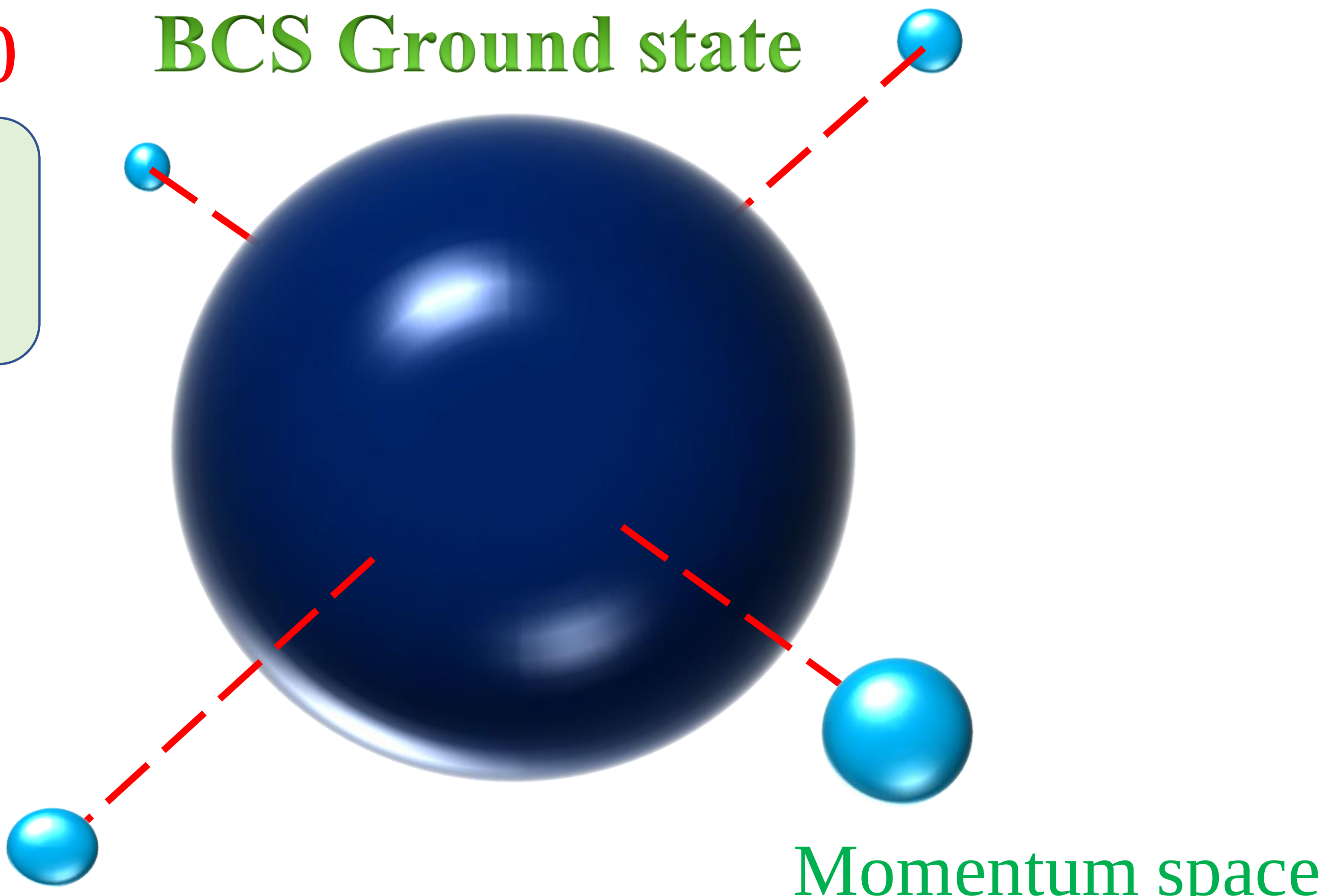
Ground-state of Fermi systems

Electrons, neutron, proton, quark, etc…

Attraction

$-U \neq 0$



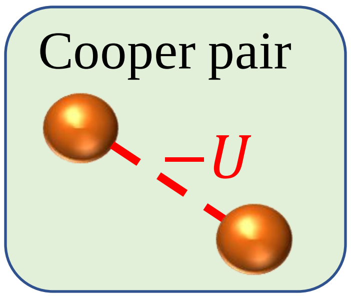


Ground-state of Fermi systems

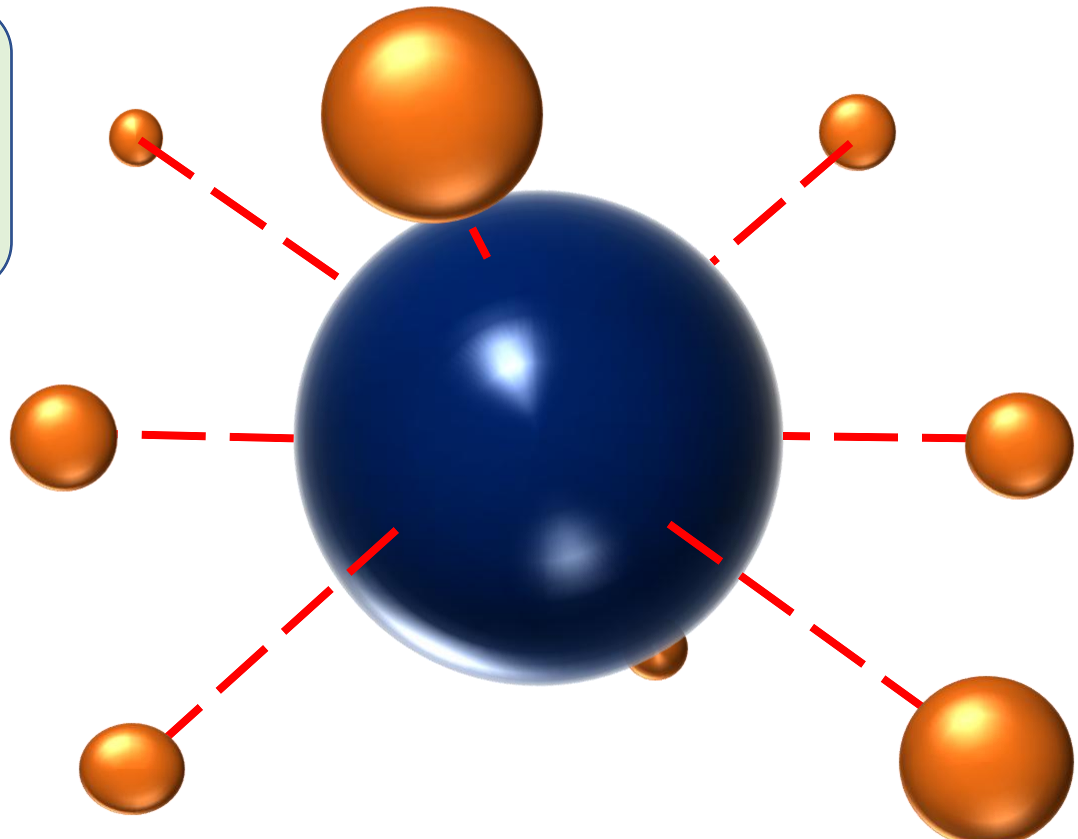
Electrons, neutron, proton, quark, etc…

Attraction

$-U \neq 0$



BCS Ground state



Momentum space

Ground-state of Fermi systems

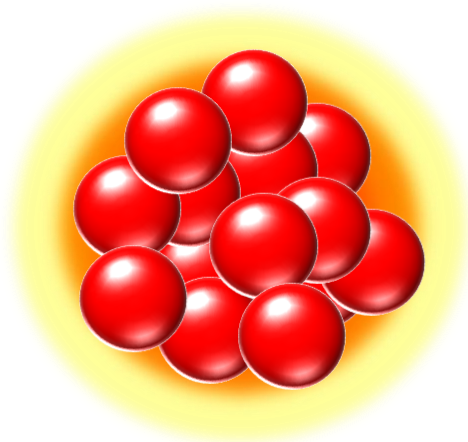
Electrons, neutron, proton, quark, etc…

Attraction

$-U \neq 0$

Molecule

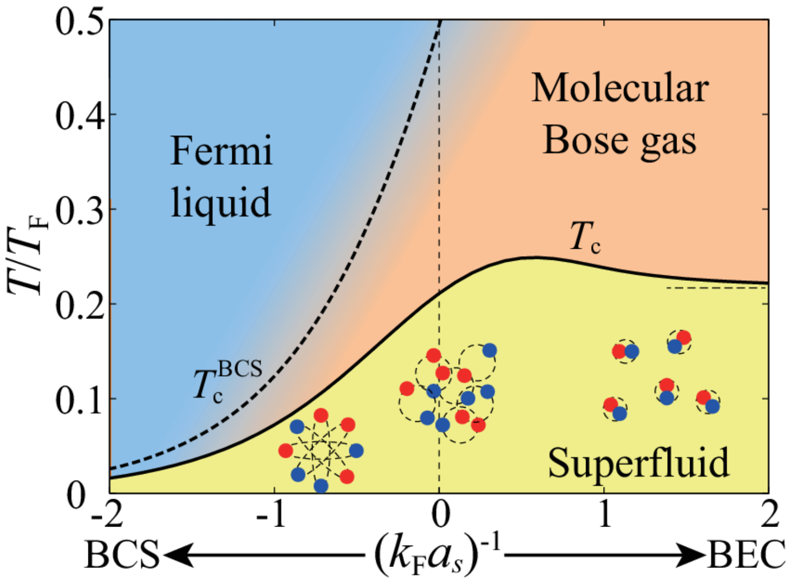
Bose-Einstein Condensate (BEC)



Superfluidity and BCS-BEC crossover

- BCS-BEC crossover is realized by tuning the scattering length
- Such a phenomenon can occur in neutron matter?

Phase diagram of the BCS-BEC crossover

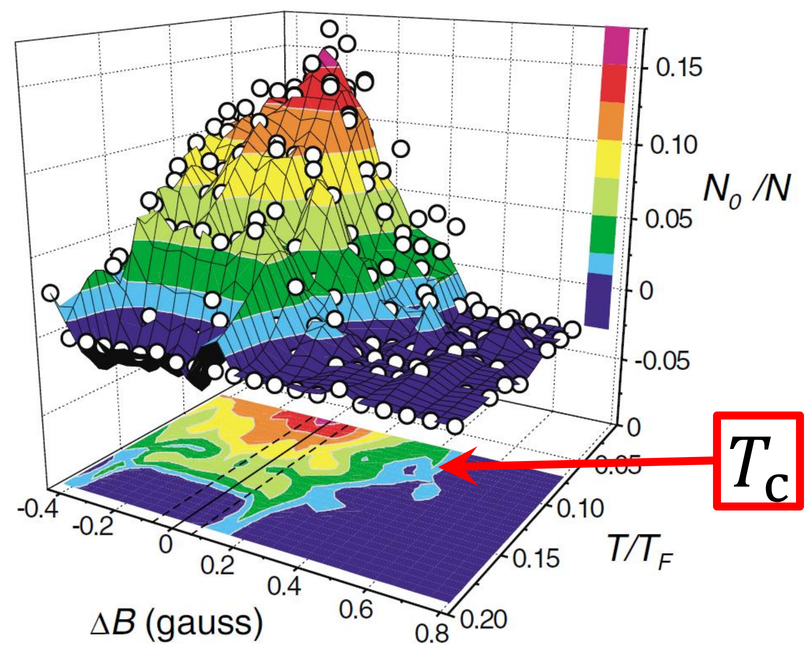


$a_s = -18.5\text{fm}$

$(k_F a_s)^{-1} \simeq 0$

http://astro.riken.jp/wordpress/?page_id=1425

Observation of BCS-BEC crossover in a ^{40}K Fermi gas



C. Regal, *et al*., PRL **92**, 040403 (2004).

Outline

- Nuclear matter and neutron star
- Ultracold atom as a "Toy model" for nuclear system
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Similarity between neutron matter and cold atom

- The low density neutron matter is also dominated by the s-wave scattering like an ultracold Fermi gas

Phase shift (effective range expansion)

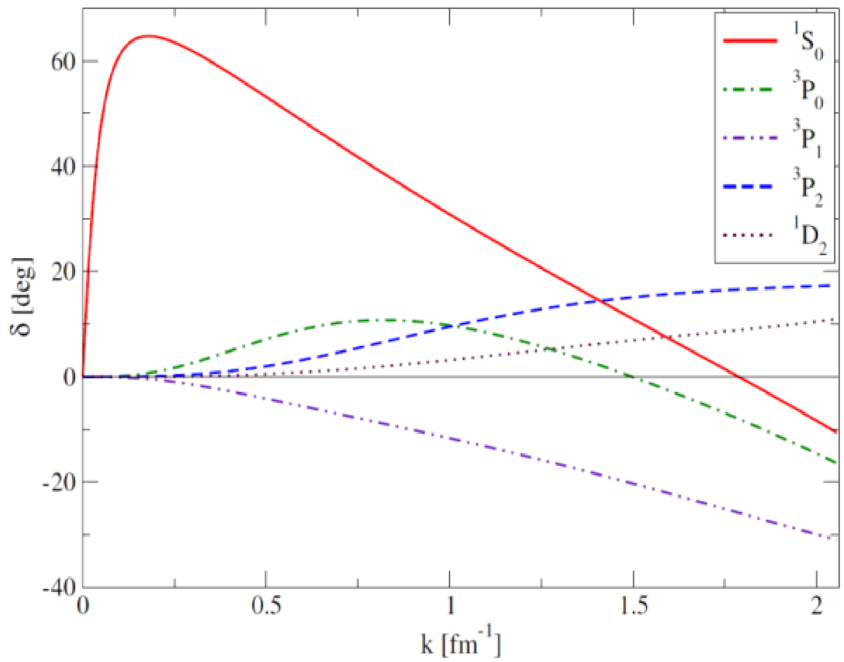
$$k\cot\delta_k = -\frac{1}{a_s} + \frac{1}{2}k^2 r_{\mathrm{eff}}$$

a_s: s-wave scattering length

r_{eff}: effective range

	Cold atom	Neutrons
a_s	$-\infty\sim\infty$	-18.5 fm
r_{eff}	~ 0	2.8 fm
density	$\sim 10^{15}\ \mathrm{cm}^{-3}$	$\sim 0.17\ \mathrm{fm}^{-3}$
$(k_{\mathrm{F}} a_s)^{-1}$	$-\infty\sim\infty$	$-\infty\sim 0$

Phase shift of NN scattering



A. Gezerlis, *et al*, arXiv : 1406.6109v2

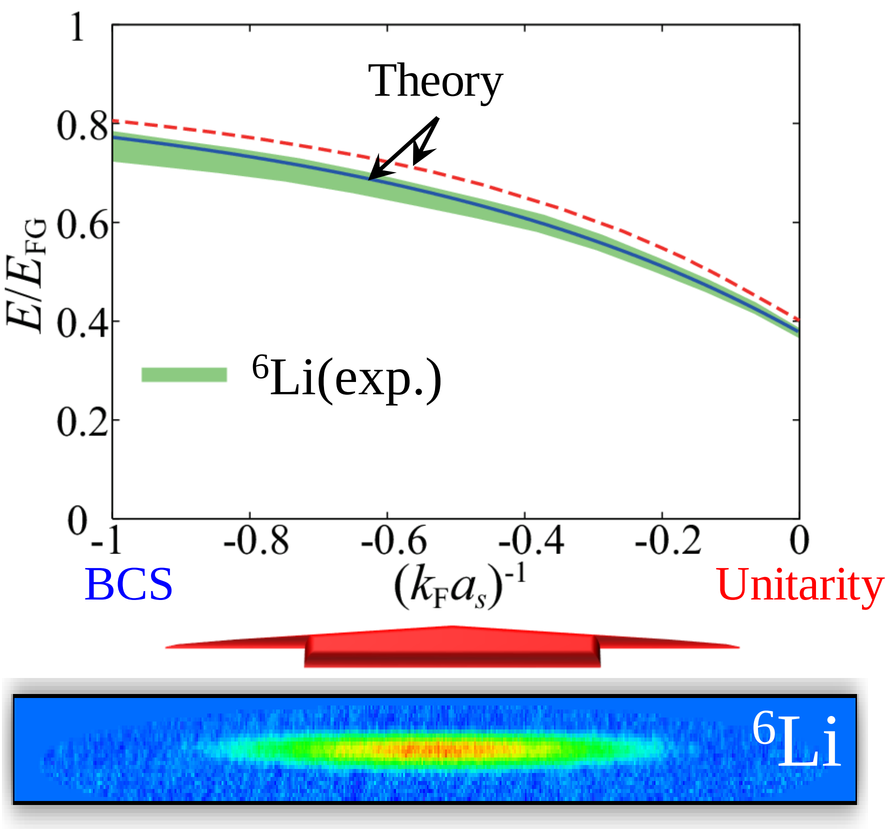
$*\ k_{\mathrm{F}}$: Fermi momentum

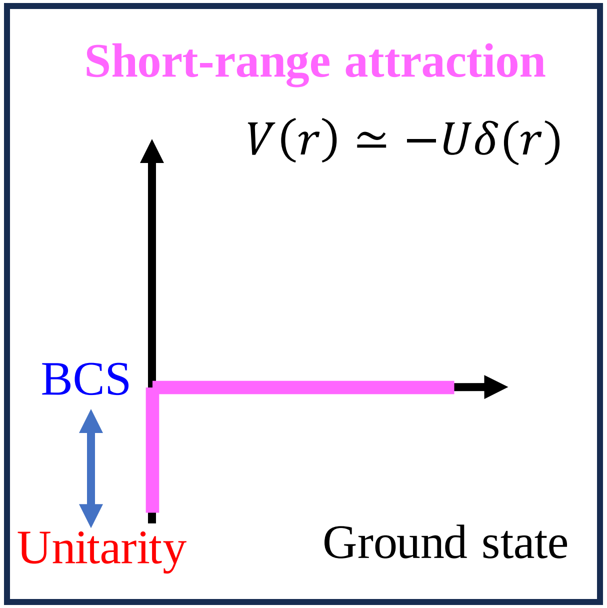
Ultracold Fermi gases
Quantum simulator for neutron matter

- The low-density neutron matter can be simulated in ultracold atomic Fermi gas experiments

M. Horikoshi, M. Koashi, HT, Y. Ohashi, and M. Kuwata-Gonokami, PRX, **7**, 041004 (2017).

Precise measurement of cold atom EOS





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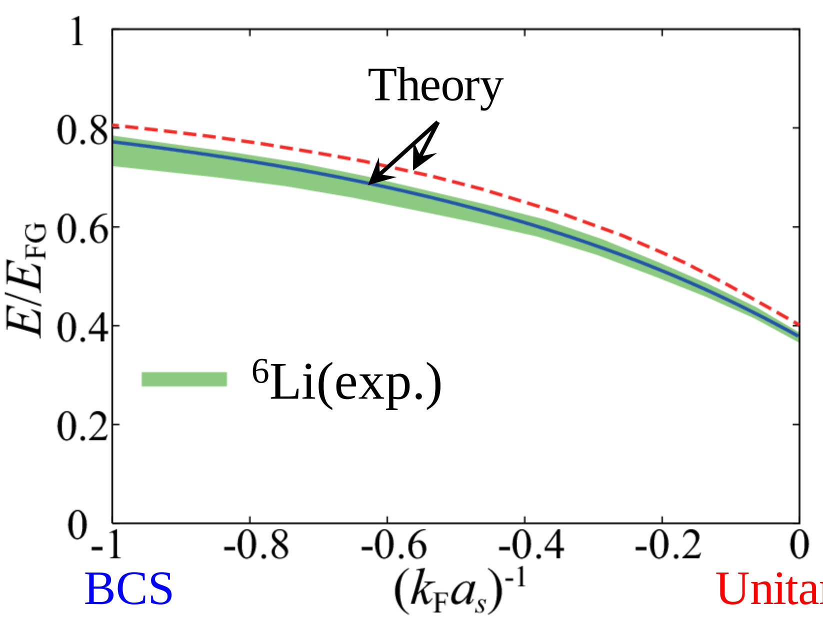
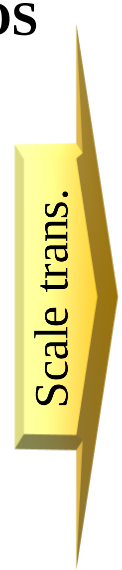
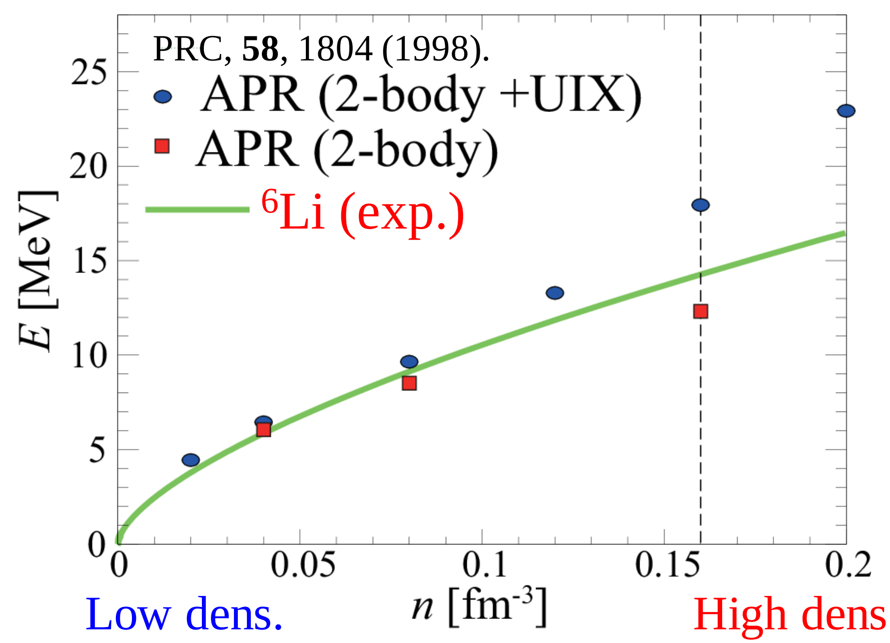
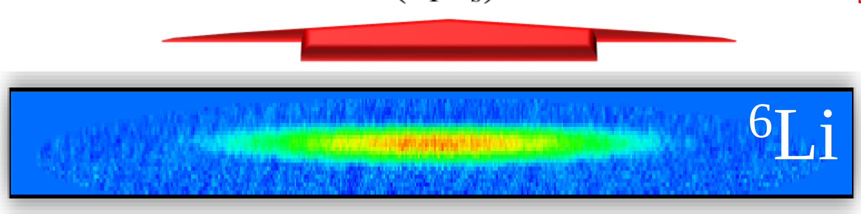




EOS of neutron matter and cold atom





Agree with each other in the low-density region

Ultracold atom side

$$\frac{E}{E_{\mathrm{FG}}} = f(x) \qquad x = \frac{1}{k_{\mathrm{F}} a_s}$$

$$E_{\mathrm{FG}} = \frac{3}{5}\rho\varepsilon_{\mathrm{F}} \quad k_{\mathrm{F}} = \left(3\pi^2\rho\right)^{1/3}$$

$\rho \sim \mathrm{nm}^{-3}$: atom density

$a_s \sim$ nm: atom-atom scattering length

Same!!!

Neutron matter side

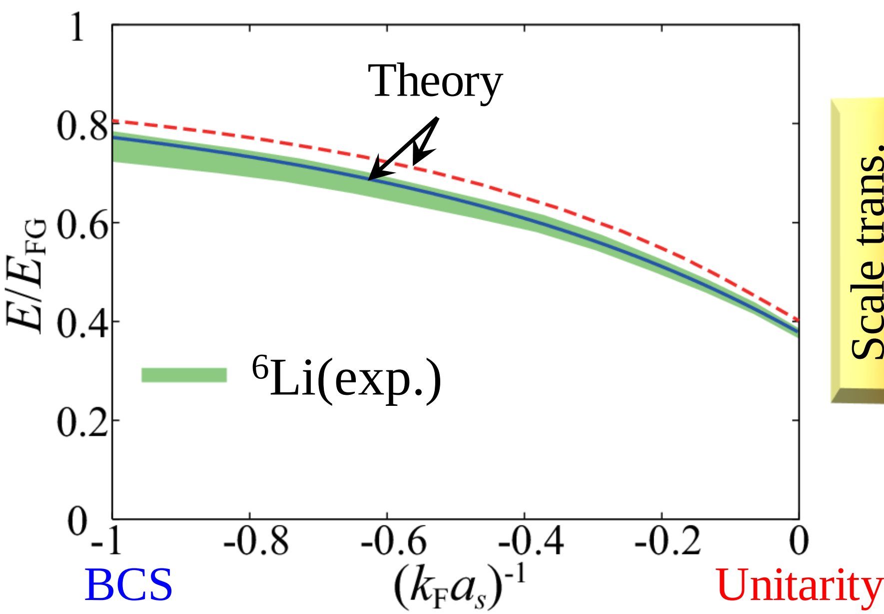
$$\frac{E}{E_{\mathrm{FG}}} = f(x) \qquad x = \frac{1}{k_{\mathrm{F}} a_s}$$

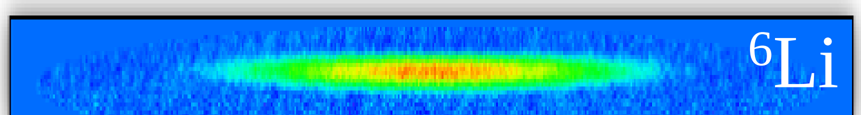
$$E_{\mathrm{FG}} = \frac{3}{5}\rho\varepsilon_{\mathrm{F}} \quad k_{\mathrm{F}} = \left(3\pi^2\rho\right)^{1/3}$$

$\rho \sim \mathrm{fm}^{-3}$: neutron density

$a_s \sim$ fm: NN scattering length

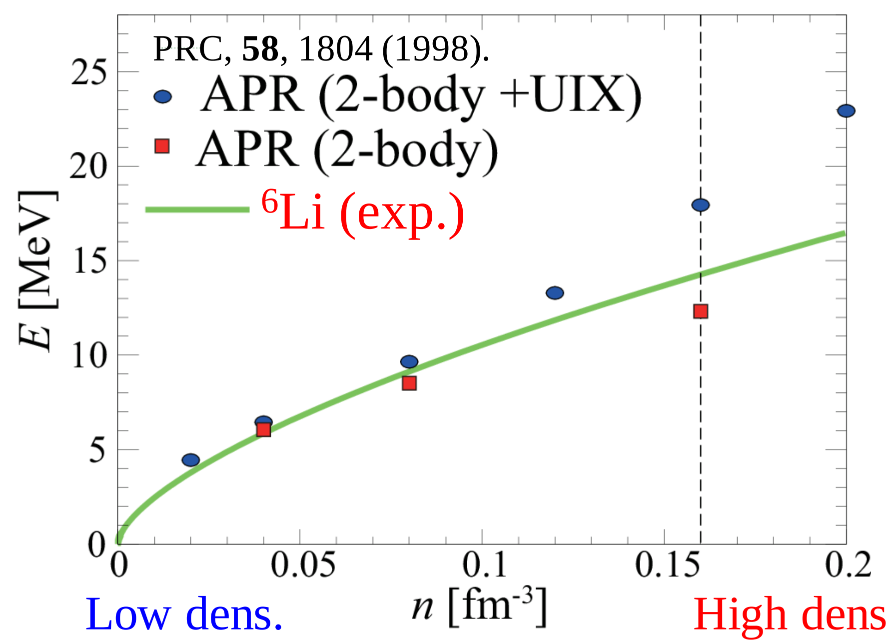
Precise measurement of cold atom EOS





Scale trans.

EOS of neutron matter and cold atom

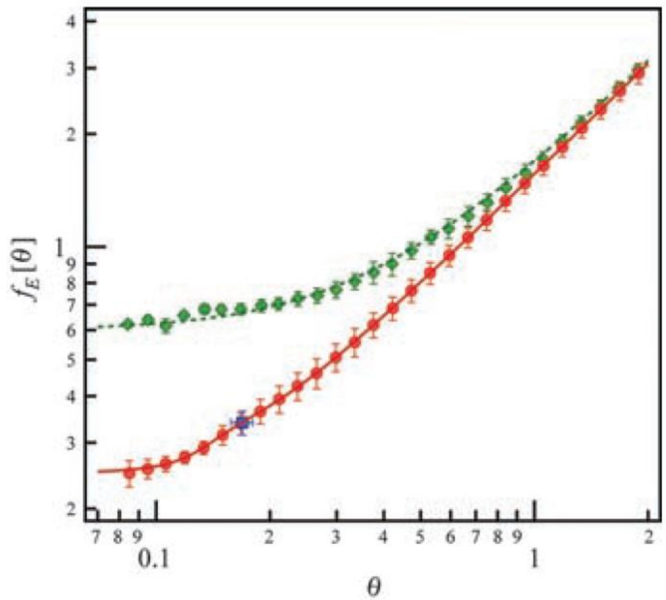


Agree with each other
in the low-density region

Unitary Fermi gas and universality

Scale invariance because of no length scales associated with interaction ($a \to \infty$)

Universal thermodynamics



M. Horikoshi, et al., Science **327**, 442 (2010).

Ideal Fermi gas: $E_{\text{ideal}} = f_0(n, T)$

Interacting Fermi gas: $E = g(n, a, T)$

Unitary Fermi gas: $E_{\text{UFG}} = f(n, T)$

At $T = 0$, only one parameter is relevant

Compressibility $\kappa = \frac{1}{\xi}\kappa_0$

Sound velocity $v_s = \sqrt{\frac{\xi}{3}}v_F$

Internal energy $E = \frac{3}{5}\xi n \varepsilon_F$

Pressure $P = \xi^{-\frac{3}{2}} P_0$

Bertsch parameter

$\xi \simeq 0.37$

G. F. Bertsch, Challenge problem in many-body physics (1999)

Unitary Fermi gas and universality

SOVIET PHYSICS JETP VOLUME 11, NUMBER 4 OCTOBER, 1960

THE EXISTENCE OF NEW ISOTOPES OF LIGHT NUCLEI AND THE EQUATION OF STATE OF NEUTRONS

Ya. B. ZEL'DOVICH

Submitted to JETP editor October 22, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 1123-1131 (April, 1960)

Just few years after BCS and Bohr-Mottelson-Pines!

For $\omega > \omega_0$, $E_f \gg E_V$, the scattering does not depend on the length a, and we can set $a = \infty$, $a^{-1} = 0$, i.e., consider resonance scattering. Then the problem contains no dimensionless parameters. From dimensional considerations it follows that in this region

$$E_1 \sim U_1 \sim \bar{E} \sim E_f \sim \omega^{2/3}. \qquad (13)$$

ω: density

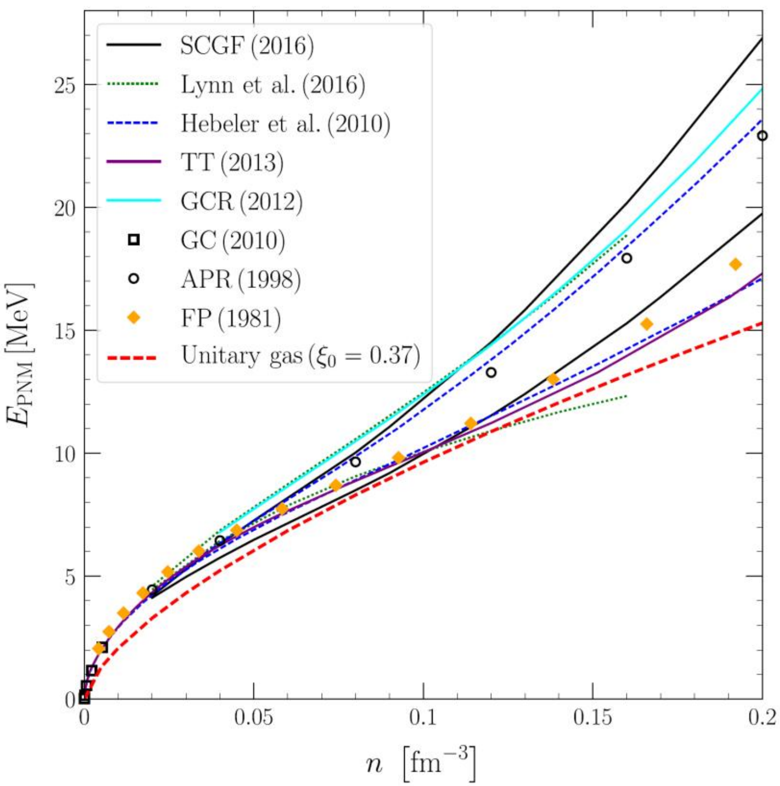
D. T. Son

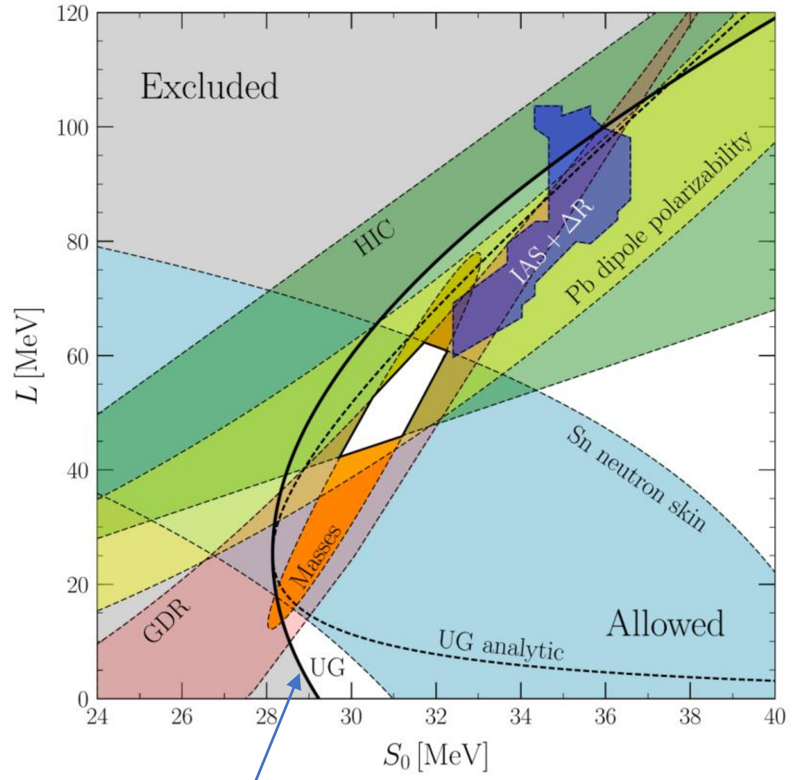
This is the first paper for a unitary Fermi gas (on FB)

Unitary gas bound conjecture

I. Tews, et al., ApJ 848:105, (2017).

Conjecture: Pure neutron matter EOS would be larger than that of a unitary Fermi gas





UG bound conjecture

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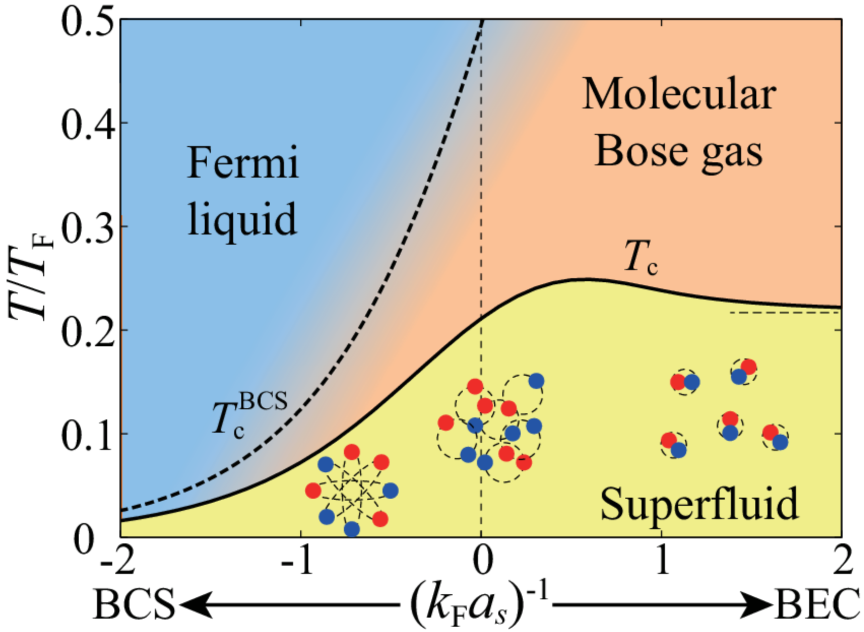
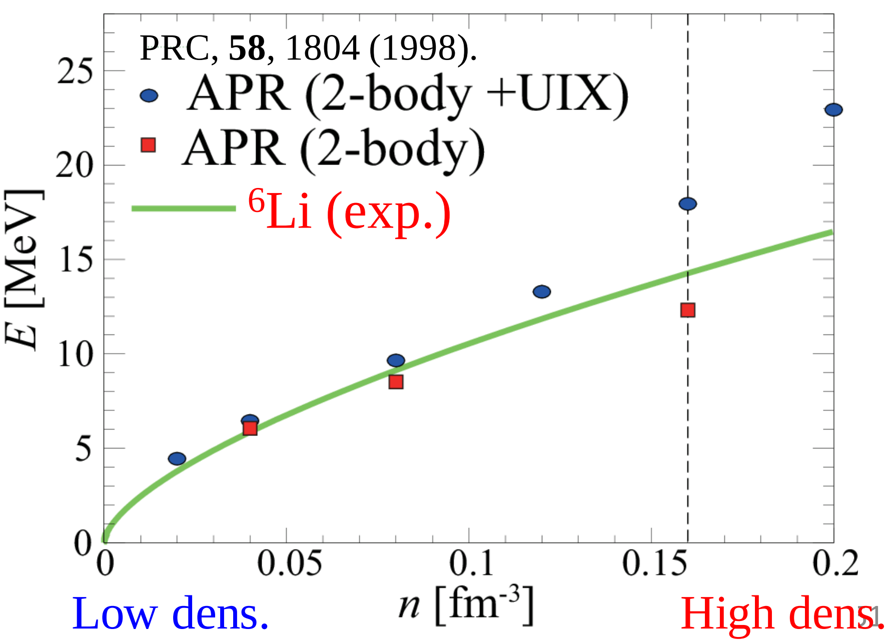
Short summary of Part 1

- Quantum many-body problems of nucleons, relevant to neutron star physics and nuclei, can be studied (at least partially) in ultracold atoms.
- In particular, pairing phenomena in an ultracold Fermi gas could be useful reference for understanding nucleon superfluids.

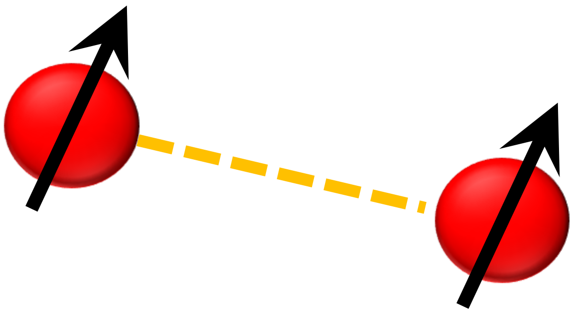




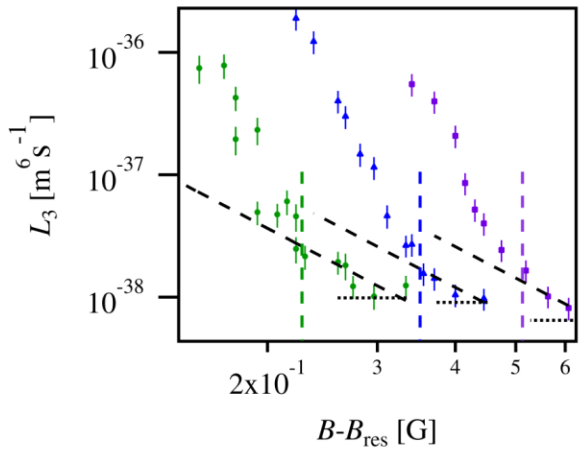
Appendix

P-wave Feshbach resonance

Spin-triplet *P*-wave pairing

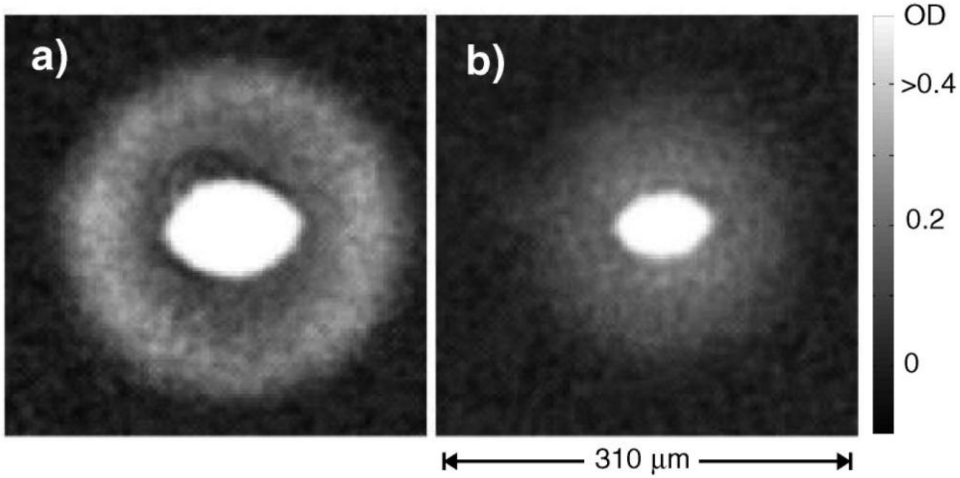


Strong three-body loss



J. Yoshida, et al.,
PRL **120**, 133401 (2018).

Observation of *p*-wave Feshbach molecule

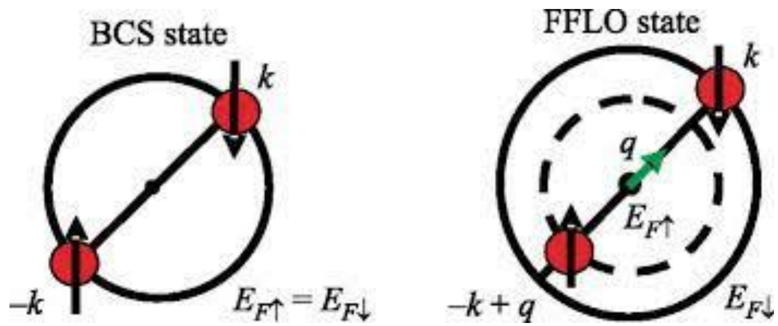


J. P. Gaebler, *et al*., PRL **98,** 200403 (2007)

P-wave Fermi superfluidity has not been realized yet due to atomic losses

Can FFLO phase exist?

Fulde-Ferrell-Larkin-Ovchinnikov state: Pair condensation at nonzero COM momentum

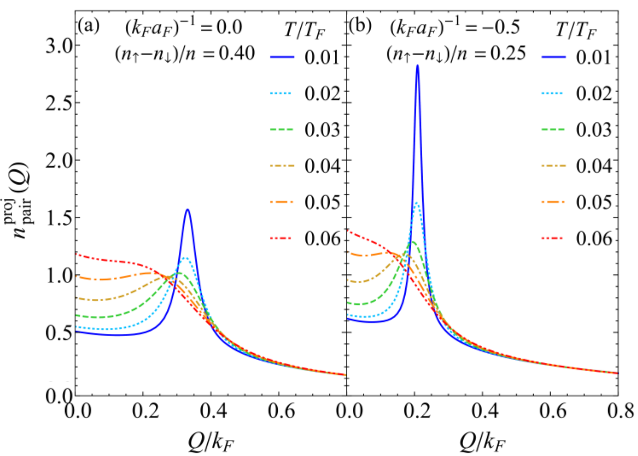


Low Temp. Phys. **39**, 225 (2013).

- Reported that it is unstable against pairing fluctuations in the homogeneous system (Y. Ohashi, JPSJ **71**, 2625 (2002).)

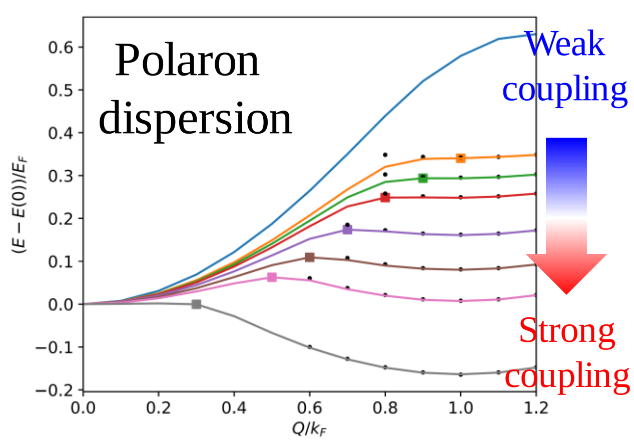
FFLO pairing fluctuations

M. Pini, et al., PRR **3**, 043068 (2021).



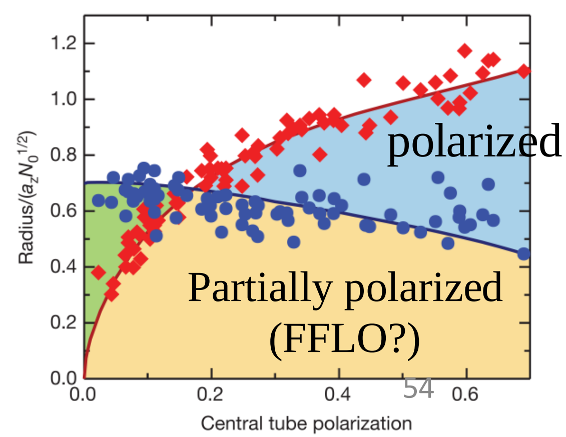
Molecule with COM momenta

C. Peng, et al., PRA **103**, 063312 (2021).



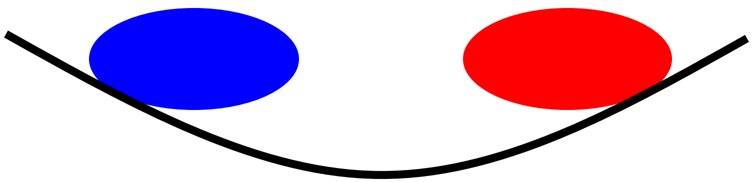
1D phase diagram (trapped)

Y. Liao, *et al.*, Nature **467**, 567 (2010).



Spin susceptibility and spin dynamics

Spin dipole oscillation



G. Valtolina, et al., Nat. Phys. **13**, 704 (2017).

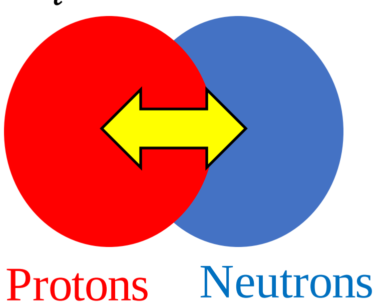
Giant dipole resonance (GDR)

Isovector GDR

$$\sum_i \boldsymbol{r}_i Y_1(\boldsymbol{r}_i)\tau_{z,i}$$

Gamov-Teller transition

$$\sum_i \hat{\boldsymbol{\sigma}}_i \tau_{\pm,i}$$



Spin-dipole resonance

$$\sum_i \boldsymbol{r}_i [Y_1(\boldsymbol{r}_i) \otimes \hat{\boldsymbol{\sigma}}_i]_\lambda \tau_{\pm,i}$$

$\tau_{x,y,z}$: isospin

Spin dipole operator

$$D_z = \sum_i z_i \hat{\sigma}_{z,i} \equiv \sum_i \boldsymbol{r}_i [Y_1(\boldsymbol{r}_i) \otimes \hat{\boldsymbol{\sigma}}_i]_{\lambda=1}$$

Energy-weighted sum rule (f-sum rule)

$$M_k = \int_0^\infty d\omega\, \omega^k S_D(\omega)$$

$S_D(\omega)$:dynamical structure factor of D_z

$$M_1 = \frac{N}{2m} \qquad M_{-1} = \frac{1}{2}\int d^3\boldsymbol{r}\, z^2 \chi(\boldsymbol{r})$$

N :total number $\chi(\boldsymbol{r})$: local spin susceptibility

Spin dipole frequency $\boldsymbol{\omega}_{\mathbf{SD}}$

$$S_D(\omega) \sim \delta(\omega - \omega_{\mathrm{SD}})$$

$$\omega_{\mathrm{SD}}^2 = \frac{N}{m \int d^3\boldsymbol{r} z^2 \chi(\boldsymbol{r})}$$

Spin dipole oscillation

In repulsive Fermi gases, the spin susceptibility diverges (spin dipole frequency becomes zero)

$$\chi = \frac{\chi_0}{1 - U\chi_0}$$

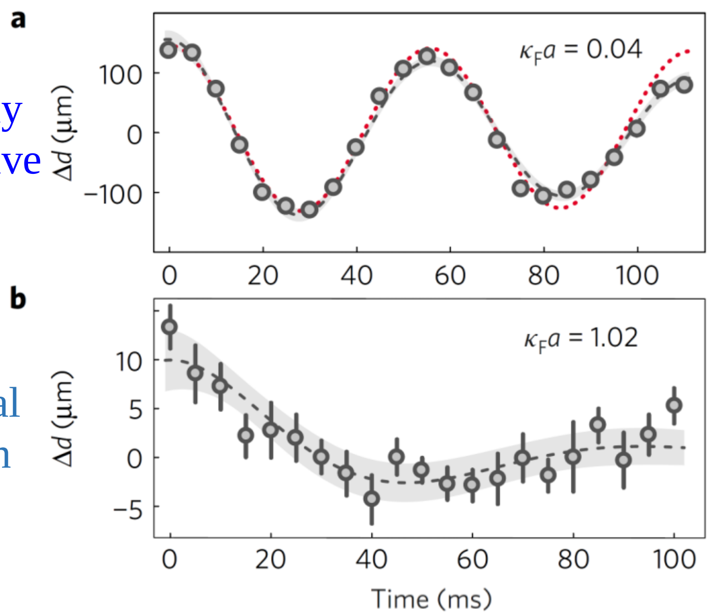
χ_0 :lowest-order particle-hole bubble

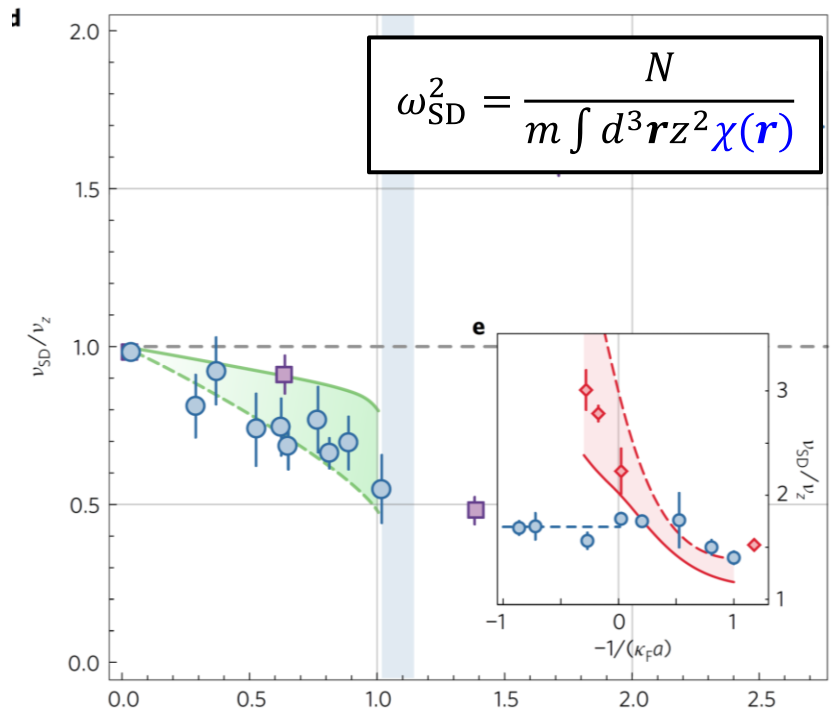
Stoner ferromagnetic instability: $1 - U\chi_0 = 0$

Observed spin dipole frequency in cold atoms

Weakly repulsive

Critical region





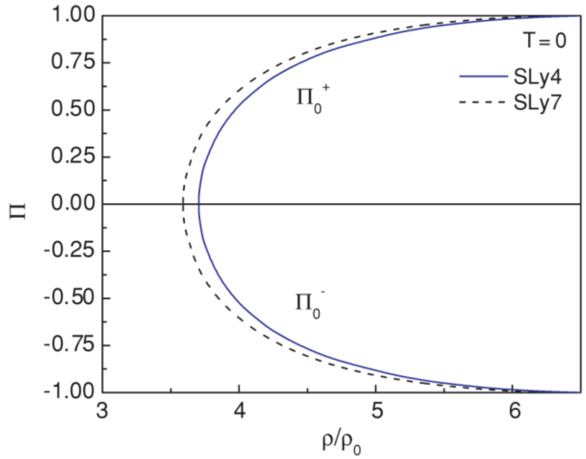
$$\omega_{SD}^2 = \frac{N}{m \int d^3\mathbf{r} z^2 \chi(\mathbf{r})}$$

Weakly repulsive $k_F a$ Strongly repulsive

G. Valtolina, et al., Nat. Phys. **13**, 704 (2017).

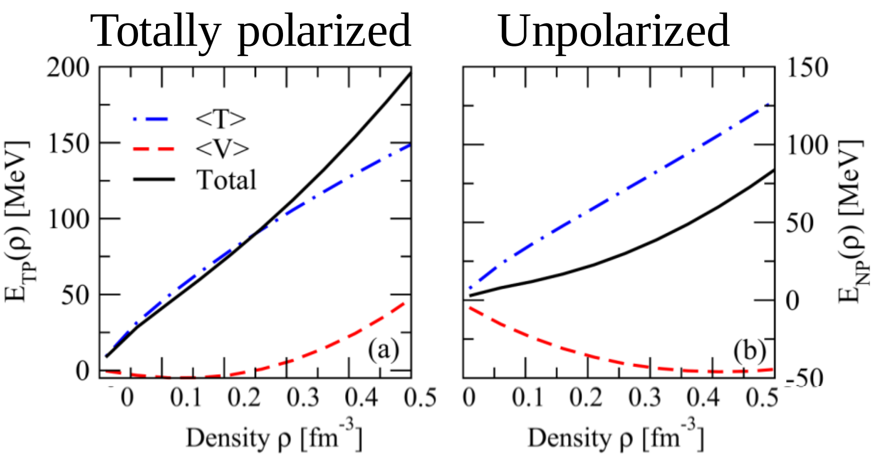
Stoner ferromagnetism in neutron matter?

Skyrme energy density functional



A. A. Isaev and J. Yang, PRC **80**, 065801 (2009).

Brueckner Hartree-Fock calculation



I. Vidana, et al., PRC **94**, 054006 (2016)

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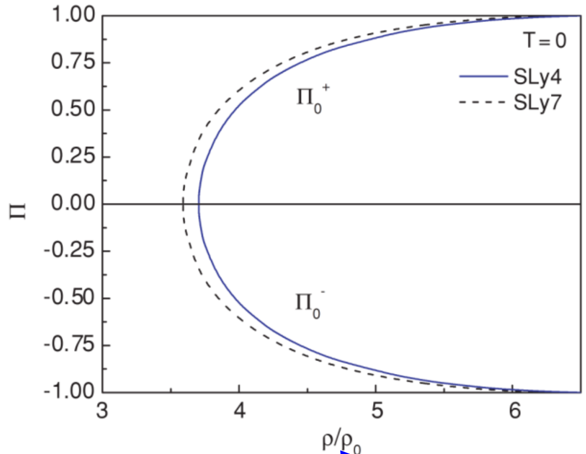
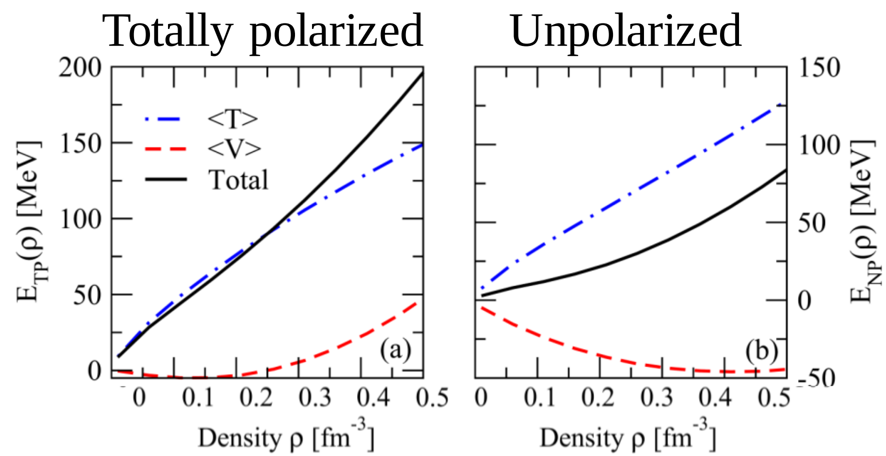


A. A. Isaev and J. Yang, PRC **80**, 065801 (2009).

YES! FM occurs at $\rho/\rho_0 \simeq 3.7$ but the framework is reliable only near $\rho/\rho_0 \simeq 1$

Brueckner Hartree-Fock calculation



I. Vidana, et al., PRC **94**, 054006 (2016)

Naïve expectation from Stoner FM

Original Stoner FM

$$k_{\mathrm{F}} a = \frac{\pi}{2}$$

$a \to r_c \simeq 0.6$ fm

r_c: repulsive core

Critical FM density

$$\rho_{\mathrm{FM}} = \frac{1}{3\pi^2}\left(\frac{\pi}{2r_c}\right)^3 \simeq 3.78\rho_0$$

Spin dipole frequency

In repulsive Fermi gases, the spin susceptibility diverges (spin dipole frequency becomes zero)

$$\chi = \frac{\chi_0}{1 - U\chi_0}$$

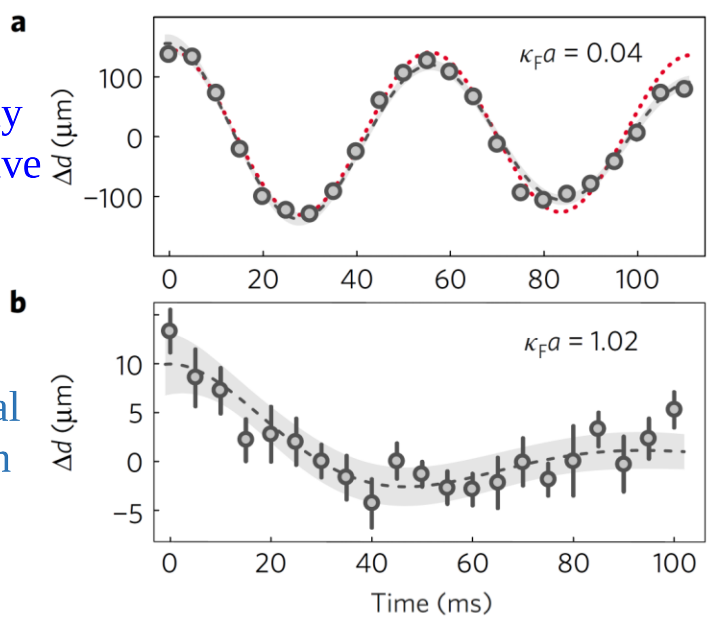
χ_0 :lowest-order particle-hole bubble

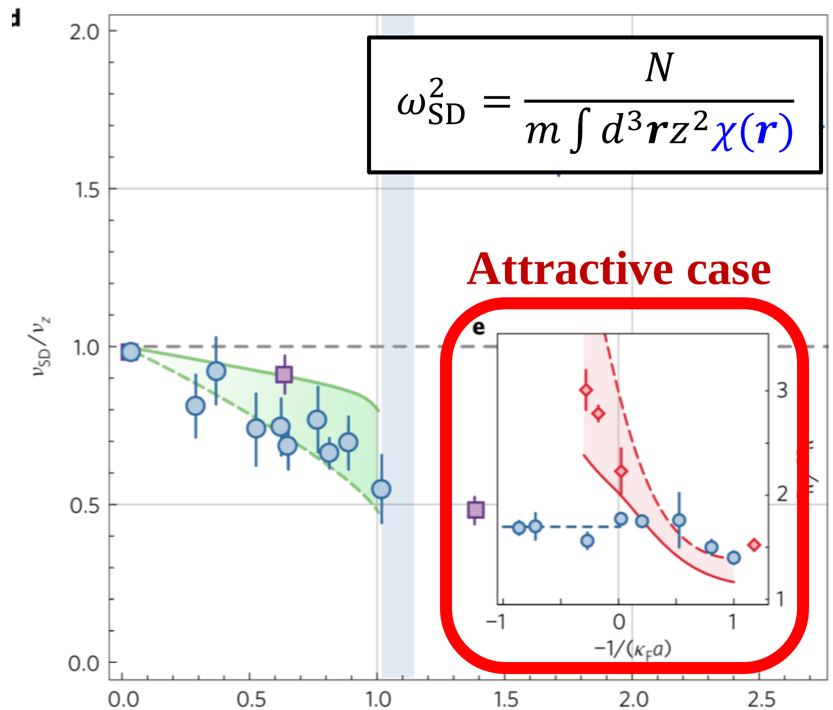
Stoner ferromagnetic instability: $1 - U\chi_0 = 0$

Observed spin dipole frequency in cold atoms

Weakly repulsive

Critical region





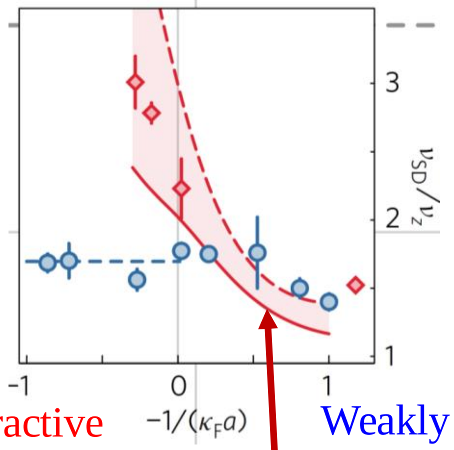
Weakly repulsive $k_F a$ Strongly repulsive

G. Valtolina, et al., Nat. Phys. **13**, 704 (2017).

Spin dipole frequency

Suppressed spin susceptibility (spin gap) leads to enhanced spin dipole frequency

Attractive case



Strongly attractive

Weakly attractive

HT+, PRA **93**, 013610 (2016)

Spin dipole frequency

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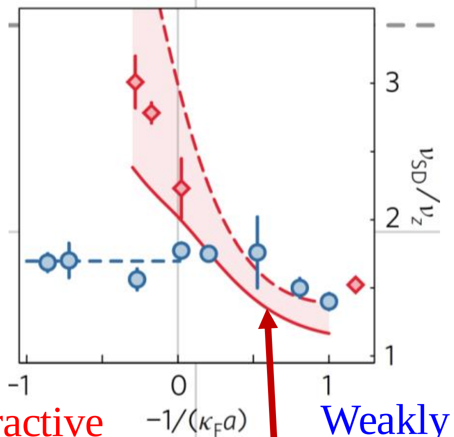
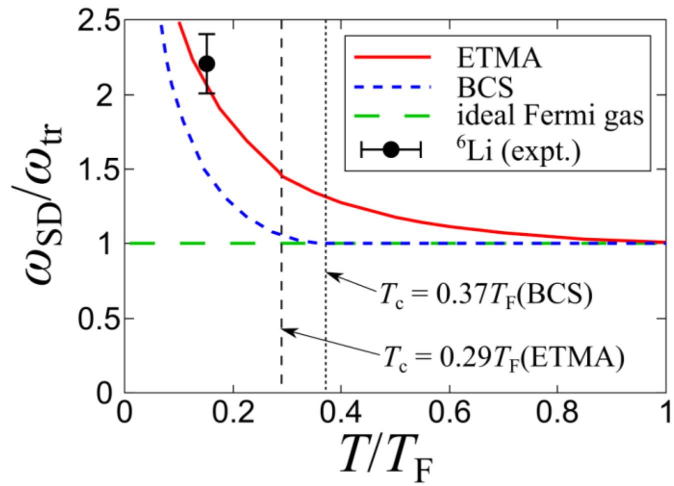


Strongly attractive

Weakly attractive

HT+, PRA **93**, 013610 (2016)

Diagrammatic approach +LDA



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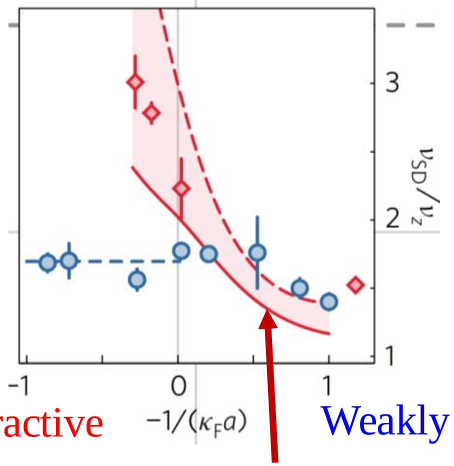
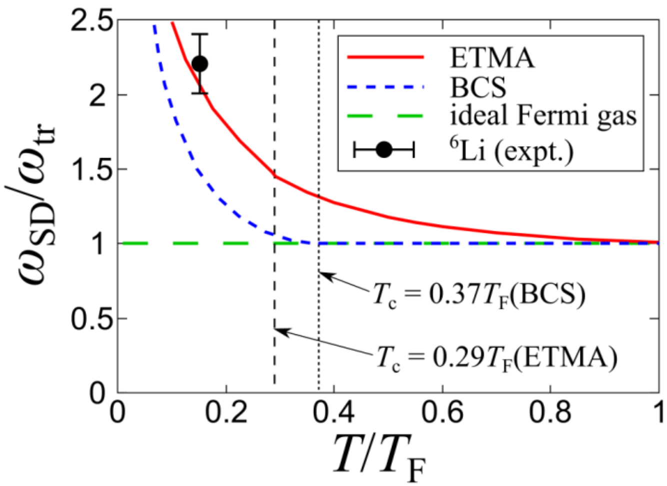


Strongly attractive　　　　Weakly attractive

HT+, PRA **93**, 013610 (2016)

Diagrammatic approach +LDA

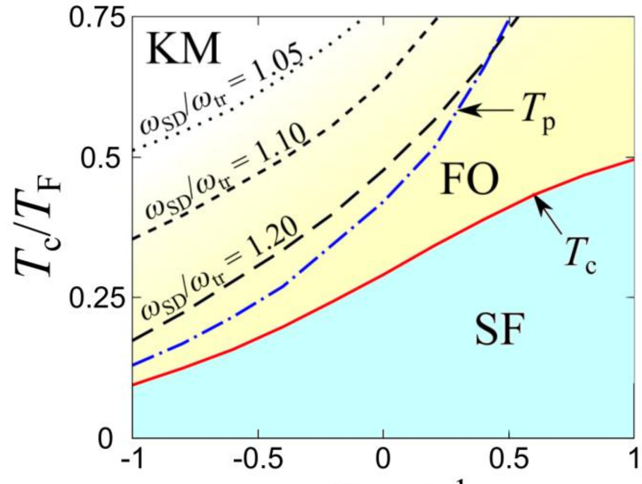


Phase diagram in terms of spin dipole frequency

・At low T, the spin dipole mode shows the fast oscillation (FO) due to the spin-singlet pairing.

・At high T, the spin dipole frequency approaches the trap frequency (Kohn mode, KM)

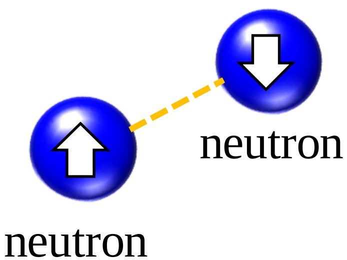
HT+, PRA **101**, 013610 (2020)



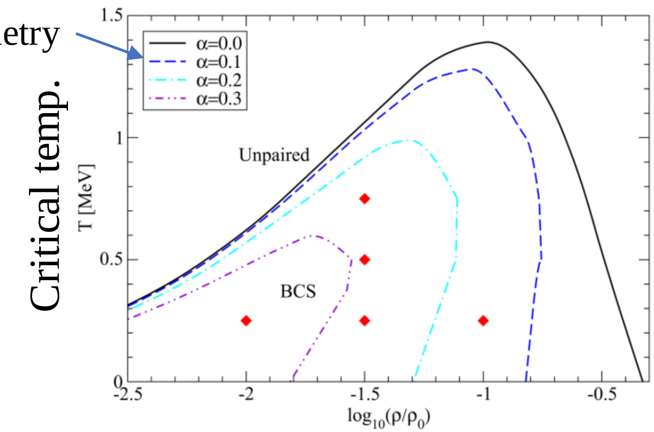
Weakly attractive　　$(k_F a_s)^{-1}$　　Strongly attractive

Pairing in “imbalanced” nuclear matter

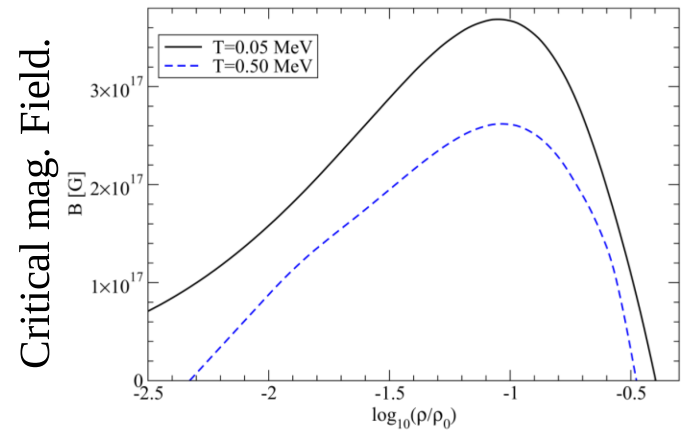
Mean-field study in spin-polarized neutron matter



α: spin asymmetry

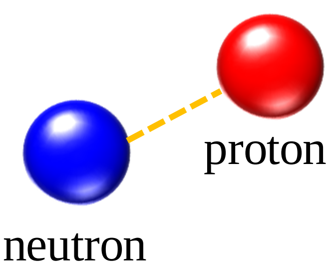


Neutron density



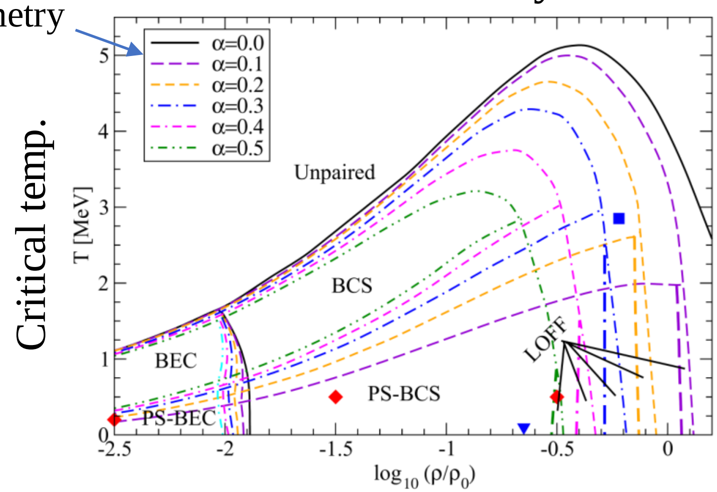
Neutron density

Spin-triplet neutron-proton pairing in asymmetric nuclear matter



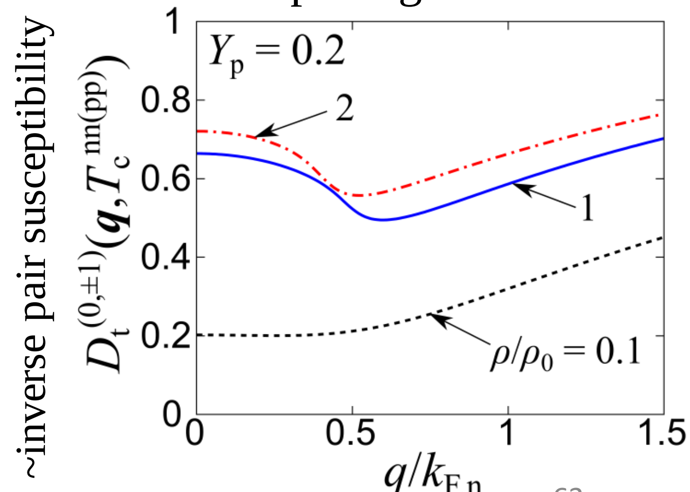
Mean-field study

α: isospin asymmetry



M. Stein+, PRC **90**, 065804 (2014).

FFLO pairing fluctuations

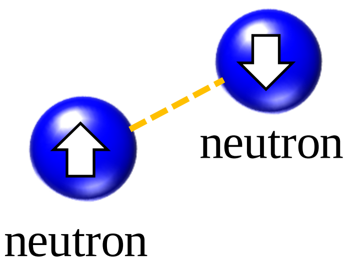


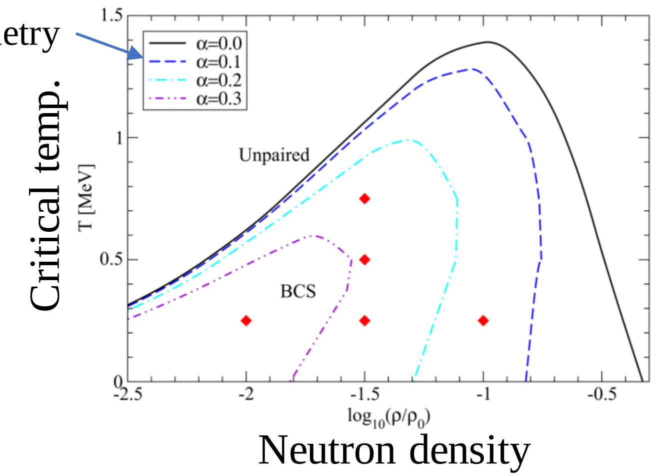
HT+, Sci. Rep. **9**, 18477 (2019).

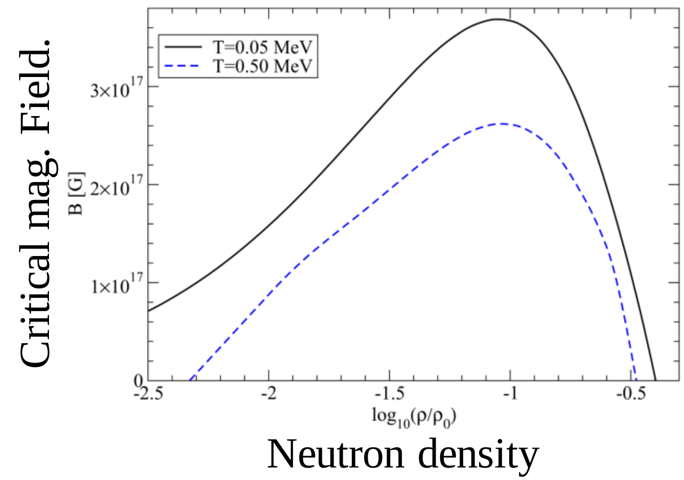
Pairing in “imbalanced” nuclear matter

Mean-field study in spin-polarized neutron matter

α: spin asymmetry





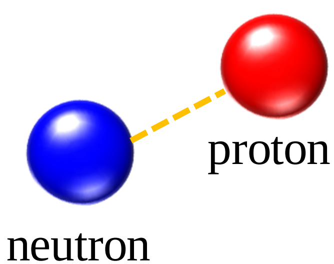


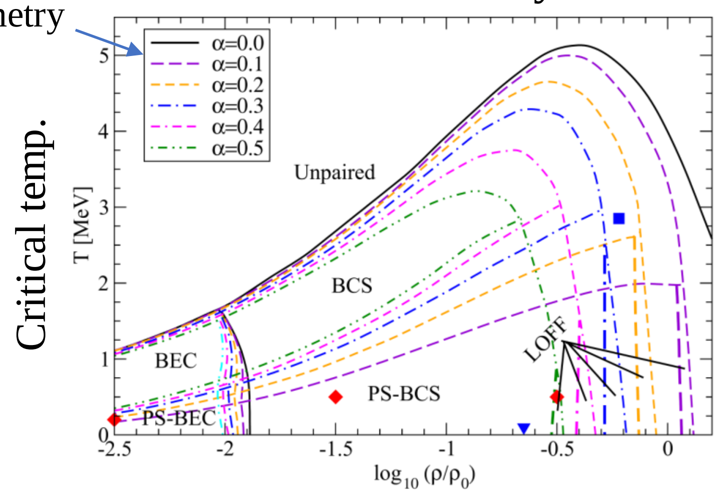
Spin-triplet neutron-proton pairing in asymmetric nuclear matter

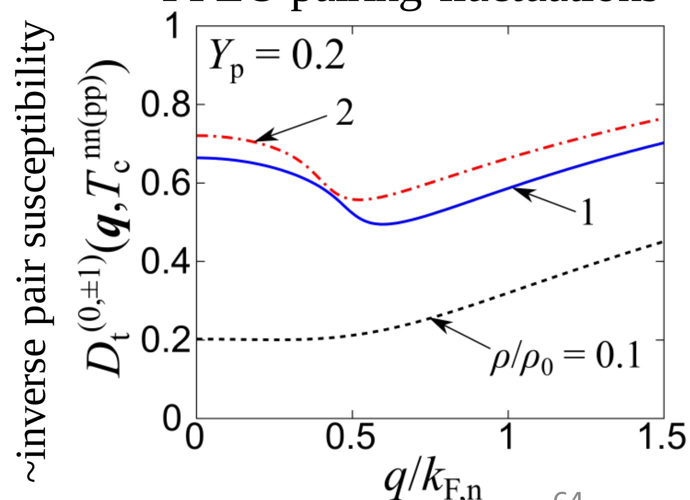
Mean-field study

FFLO pairing fluctuations

α: isospin asymmetry







M. Stein+, PRC **90**, 065804 (2014).

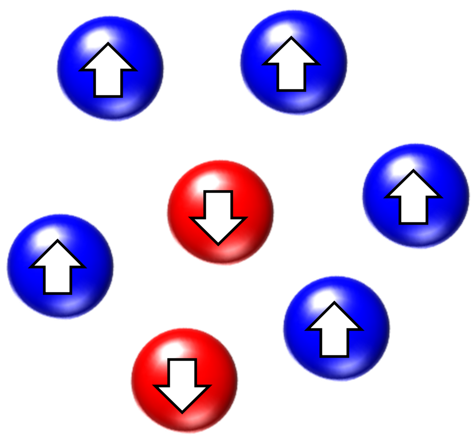
HT+, Sci. Rep. **9**, 18477 (2019).

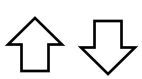
Population-imbalanced Fermi gases

“Mimicking” magnetized neutron matter

Atomic numbers $N_{\uparrow,\downarrow}$ can also be tunable (chemical potential $\mu_{\uparrow,\downarrow}$)

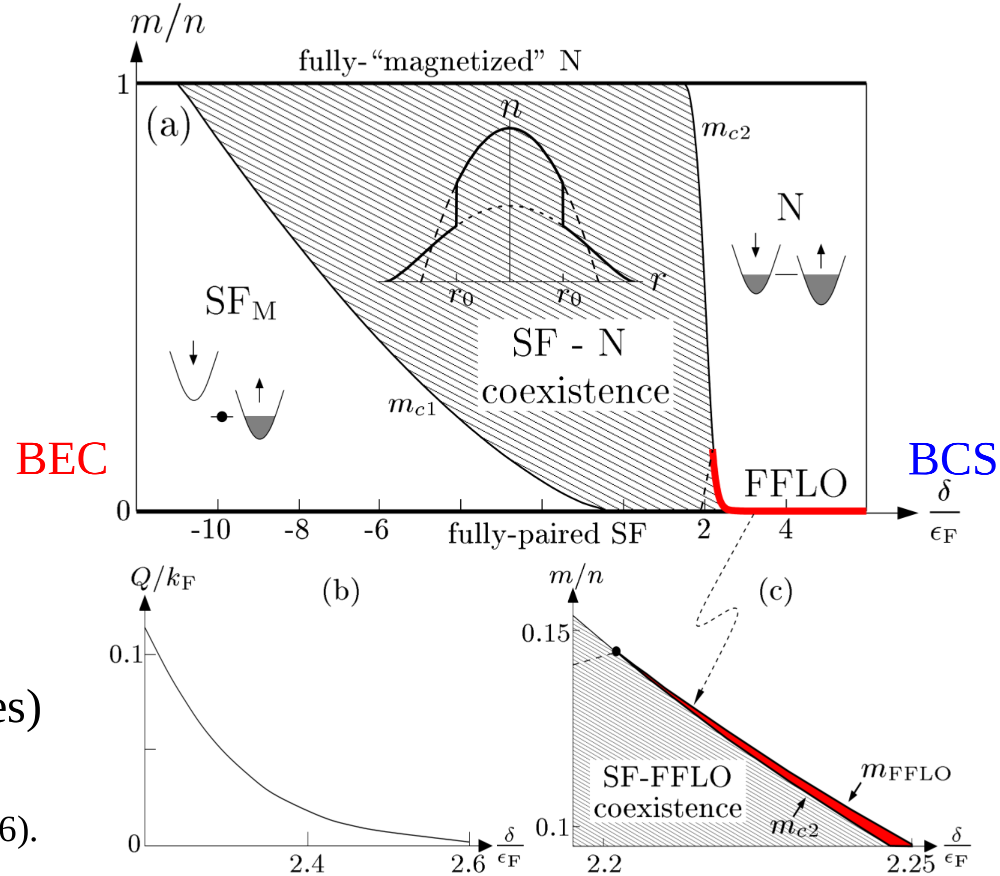
Fictitious Zeeman field: $h = \mu_{\uparrow} - \mu_{\downarrow}$



 : pseudospin (hyperfine states)

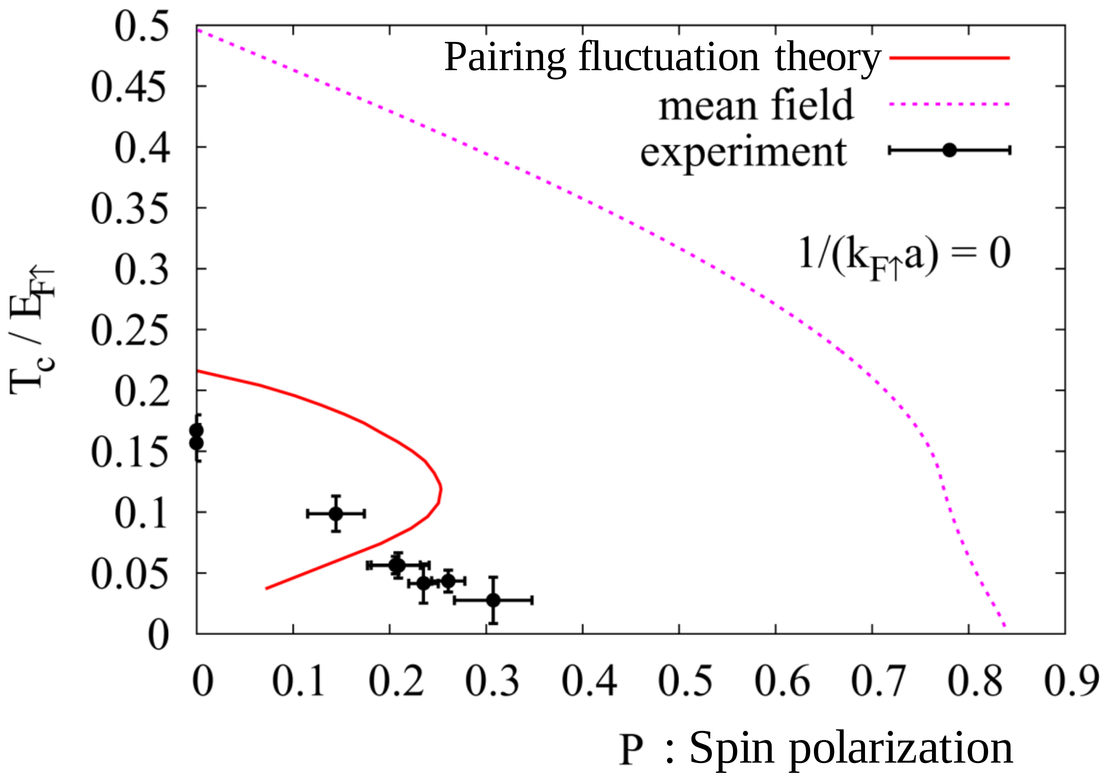
D. E. Sheehy and L. Radzihovsky, PRL **96**, 060401 (2006).

Mean-field phase diagram at *T* = 0



Beyond mean field theory

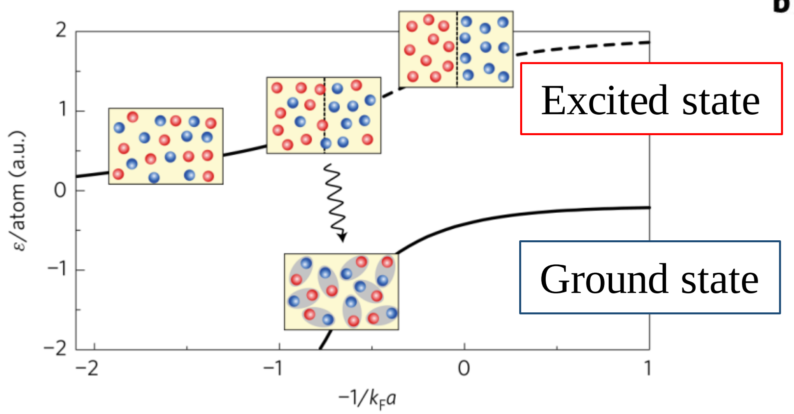
Mean-field theory overestimates the critical temperature and spin polarization

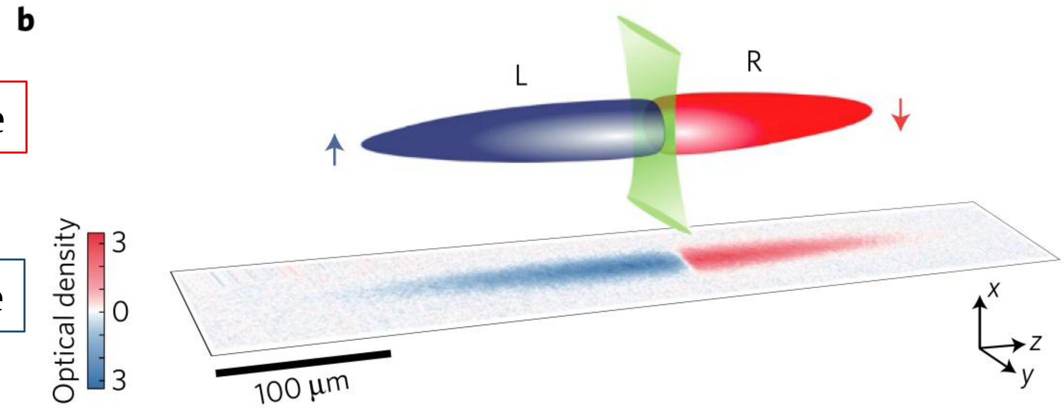


P.-A. Pantel, D. Davesne, and M. Urban, PRA **90**, 053629 (2014)

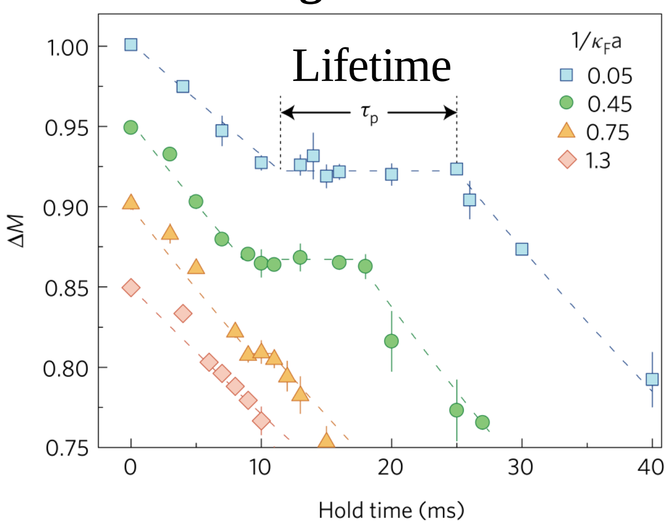
Repulsive Fermi gases: Metastable Stoner ferromagnetism

G. Valtolina, et al., Nat. Phys. **13**, 704 (2017).

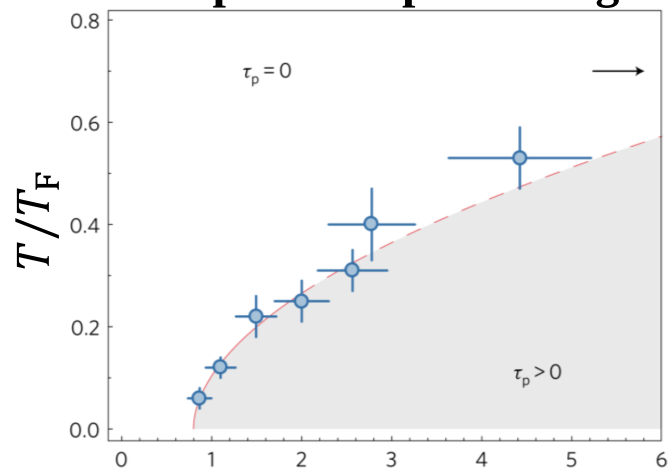




Magnetization



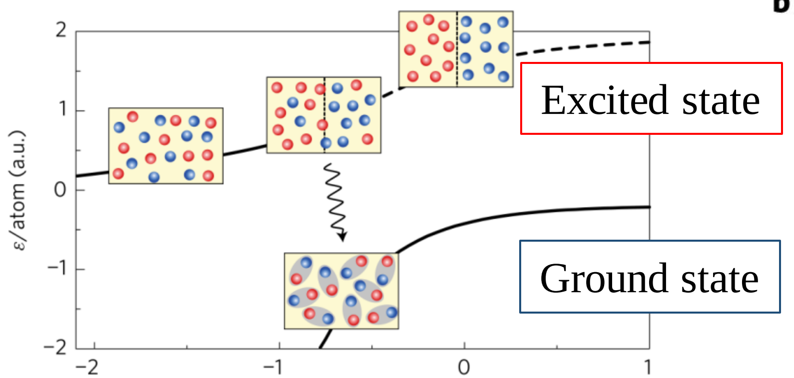
Finite temperature phase diagram

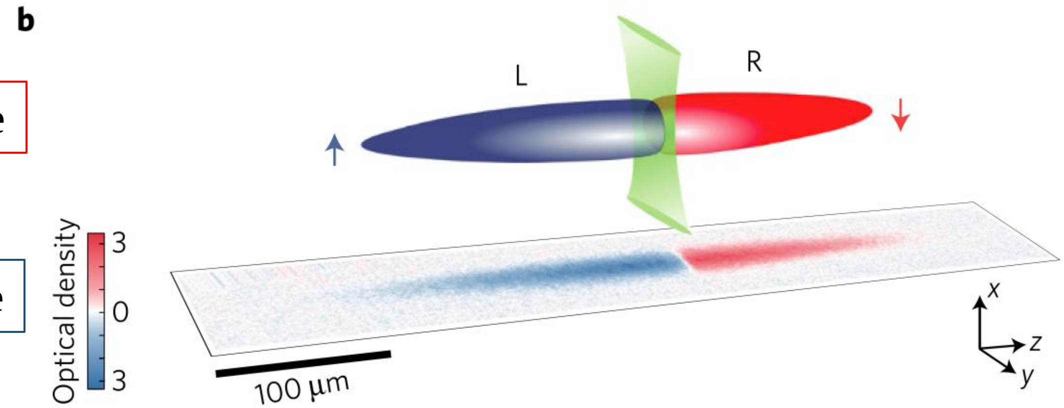


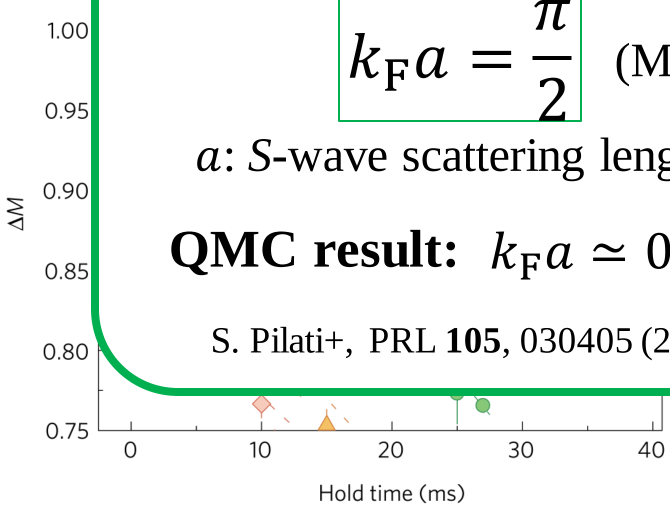
Weakly repulsive $k_F a$ Strongly repulsive

Repulsive Fermi gases: Metastable Stoner ferromagnetism

G. Valtolina, et al., Nat. Phys. **13**, 704 (2017).







Stoner ferromagnetism

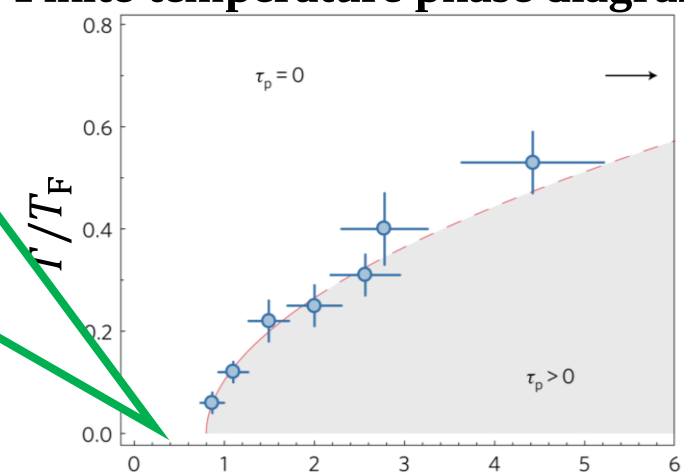
$$k_{\mathrm{F}}a = \frac{\pi}{2}$$ (Mean field)

a: S-wave scattering length

QMC result: $k_{\mathrm{F}}a \simeq 0.8$

S. Pilati+, PRL **105**, 030405 (2010)

Finite temperature phase diagram



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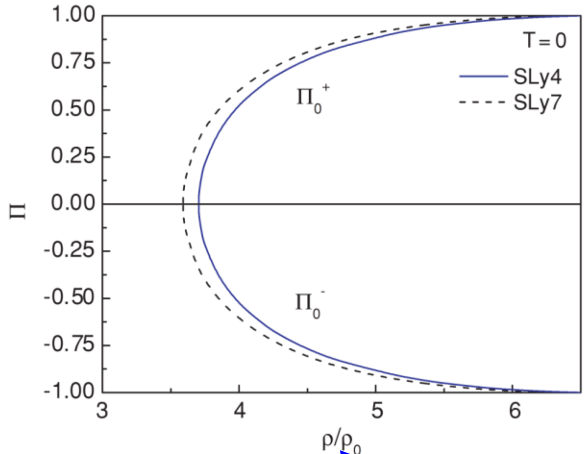
Stoner ferromagnetism in neutron matter?

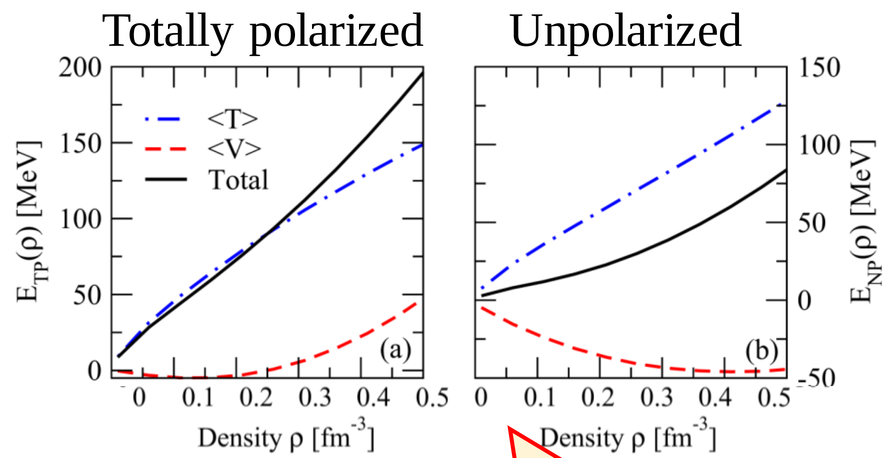
Skyrme energy density functional



A. A. Isaev and J. Yang, PRC 80, 065801 (2009).

YES! FM occurs at $\rho/\rho_0 \simeq 3.7$ but the framework is reliable only near $\rho/\rho_0 \simeq 1$

Brueckner Hartree-Fock calculation



I. Vidana, et al., PRC **94**, 054006 (2016)

No! FM never occurs with Argonne V18 and Urbana IX, but the three-body force is considered as density-dependent two-body one

Naïve expectation from Stoner FM

Original Stoner FM

$$k_F a = \frac{\pi}{2}$$

$a \to r_c \simeq 0.6$ fm

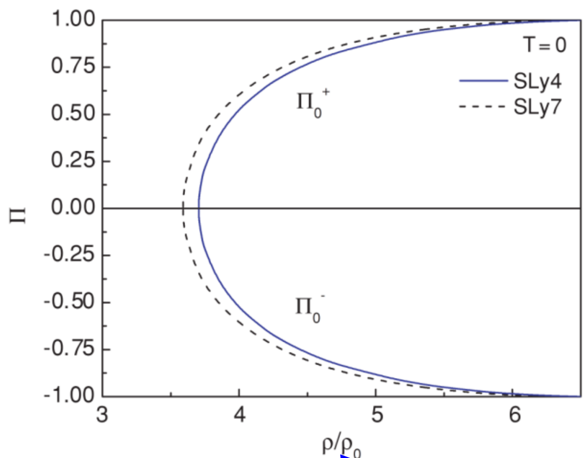
r_c: repulsive core

Critical FM density

$$\rho_{FM} = \frac{1}{3\pi^2}\left(\frac{\pi}{2r_c}\right)^3 \simeq 3.78\rho_0$$

Stoner ferromagnetism in neutron matter?

Skyrme energy density functional



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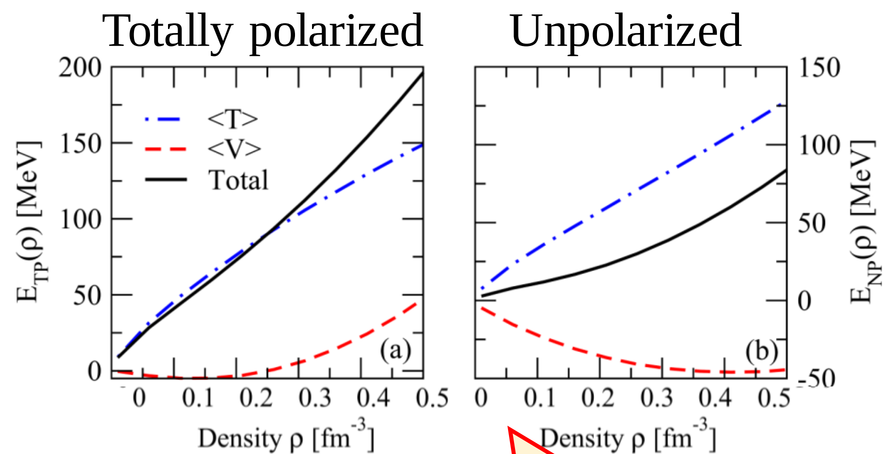

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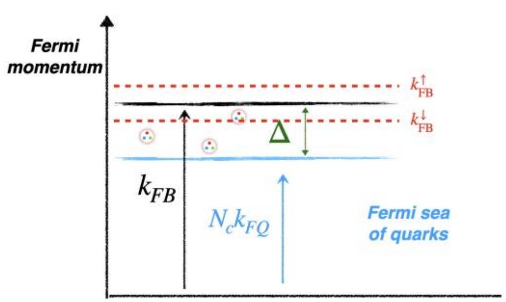

Quarkyonic matter description (w quark DOF) might be promising for this issue

B. Gao and K. Yoshida, arXiv:2507.06577